NEW MENTAL ARITHMETIC

EDWARD BROOKS, A.M., Ph.D.

THE

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PREFACE.

THE science of Arithmetic, until somewhat recently, was much less useful as an educational agency than it should have been. Consisting mainly of rules and methods of operations, without presenting the reasons for them, it failed to give that high degree of mental discipline which, when properly taught, it is so well calculated to afford. But a great change has been wrought in this respect; a new era has dawned upon the science of numbers; a "royal road" to mathematics has been discovered, so graded and strewn with the flowers of reason and philosophy that the youthful learner can follow it with interest and pleasure; and one of the most influential agents in this work has been the system of mental arithmetic.

The importance of this change can hardly be overestimated. The study of mental arithmetic, introduced by Warren Colburn, to whom teachers and pupils owe a debt of gratitude which can never be paid, affords the finest mental discipline of any study in the public schools. When properly taught, it gives quickness of perception, keenness of insight, toughness of mental fibre, and an intellectual power and grasp that can be acquired by no other elementary branch of study. An old writer on arithmetic quaintly called his work "The Whetstone of Wit;" mental arithmetic is, in my opinion, truly a *whetstone of wit.* It is a *mental grindstone;* it sharpens the mind and gives it the power of concentration and penetration. To omit a thorough course of mental arithmetic in the common school is to deprive the pupil of one of the principal sources of mental power.

Realizing these views, the author, about fifteen years ago, at the earnest solicitation of teachers and educational friends, prepared and published the Normal Mental Arithmetic. The immediate object of the work was to meet a demand made by the advance of the educational interests and the elevation of the standard of qualification of teachers of public schools for a more philosophic, systematic, and comprehensive text-book upon the subject than had hitherto been presented. This work

was received with great favor by the public, and has had a large circulation. It is a great satisfaction to the author to know that it has laid the foundation of the mathematical education of thousands of the boys and girls of this and adjoining States, and has contributed something toward improving the methods of instruction in the science of numbers.

During the past fifteen years the author has carefully noticed the working of the book in his own classes and in the public schools, with the intention of making such improvements as the wants of the school-room suggested. The improvements thus indicated are a better grading of its problems and a fuller treatment of several topics, and these are the principal changes made in this revision. In its spirit and general characteristics the book remains as when first published. All of the old favorite problems, which on account of some nice point have sometimes puzzled pupils who did not stop to think, are retained, though their position may have been changed. These problems were written for actual school-room work, when the author was interested in teaching the subject, and they therefore possess peculiarities requiring a little ingenuity of thought which make them superior to any new ones which he could now write. Most of the old forms of solution also remain; but in a few cases, as in the *age problems*, a solution is given which seems simpler than the old one.

The principles upon which the work is based, as presented when it was *first published*, will be restated.

A system of mental arithmetic should be based upon the principles of *Analysis* and *Induction*. Results should be derived by *analytic processes*, and *methods* inferred from these processes; and this is the philosophy upon which the present treatise is founded. Analysis and Induction are the golden keys which unlock the various complex combinations of numbers—the magic wand by which the intricate and abstruse are unfolded in logical simplicity.

The great element of Analysis is *comparison*, and the *equation* is the Archimedean lever of comparison. It enters into every operation, from the simplest combination of Arithmetic to the most complicated problem of the transcendental Analysis. In Geometry the axioms and definitions are the standards of comparison; in Algebra we compare the unknown with the known to determine its value; and in Arithmetic we compare all numbers, and the effects produced by a number of equal causes, with the unit, or effect of the single cause. And thus the science of Mathematics is evolved, comprising a vast series of dependent truths derived from successive comparisons of the unknown with the known, the theoretic with the axiomatic, the complex with the simple.

In the science of numbers, this relation of the collection to the unit is so evident that it is intuitively apprehended, and hence the simplicity of this elementary process of Analysis. But as the pupil progresses in the science, it will be perceived that different collections bear certain relations to each other, and he should be taught to discover and apply these new relations. To develop this theory fully and completely has been the object of the author.

The principle of analysis consists in making the *unit* the basis of the reasoning process. In analysis we regard the unit as the starting-point of numbers, and reason *to the unit* and *from the unit*. The unit thus becomes the *central point* around which the reasoning process revolves. This method of analysis and induction is especially exemplified in the treatment of fractions. In each case, after an analysis of several problems, *methods* of operation or *rules* are derived from the analysis by induction. The whole work is designed to be a complete embodiment of the spirit of analysis, which, when fully acquired by a pupil, gives him power to work independently of memory or method, and enables him to originate his own methods and form his own rules.

The work is divided into eight sections, the nature and scope of each of which will be briefly stated:

SEC. I. This section treats of the fundamental operations. The object is to make pupils ready and accurate in combining and separating numbers.

SEC. II. This section is an Introduction to Fractions, in which the *fractional word* is used, treating them as concrete numbers. The elements of *Arithmetical Analysis* are also here presented.

SEC. III. This section is a full and logical treatment of Fractions by *Analysis* and *Induction*. Special problems are solved by analysis, and rules derived from the analyses by inference or induction.

SEC. IV. This section treats of Denominate Numbers, giving the tables and sufficient exercises to make pupils familiar with them.

SEC. V. This section presents the principal cases of Proportion which may be treated by analysis. The solutions in this section exemplify the nature of arithmetical analysis.

SEC. VI. This section embraces all the principal cases of Percentage and Interest treated by the simple method of arithmetical analysis.

SEC. VII. and VIII. These sections consist of the analyses of a large number of interesting problems which, until the introduction of arithmetical analysis, were confined to algebra. Many of these problems are very old, their authors being un-

known; several of them are original with this work. The analysis will be found in many cases simpler than the algebraic solution. Sec. VII. is designed for a shorter course, when the time for the study is limited; and Sec. VIII. for a full and complete course in the science. The problems in these sections are so graded that teachers who desire can simplify the course by having pupils solve the problems in each lesson as far as the one marked with a star.

Especial attention is invited to the headings of the lessons. The object of this is to give pupils a clear idea of the nature of the subject which they are considering. In the last two sections, which consist of the analysis of distinct classes of problems, it was sometimes difficult to decide upon appropriate names for the problems. The names selected are those by which the problems are usually known among teachers and pupils. Generally some prominent peculiarity of the typical problem is taken to give the name to the class. It will be frequently noticed that, after beginning the lesson with the typical problem, variations are made both in the conditions of the question and in their application to other objects than those named in the original problem. This is done to give variety to the exercises and to afford discipline to the pupil.

The work as thus revised is presented to the public with the hope that it may retain all of its old friends and be found worthy of many new ones, and that, by the mental discipline which it will afford to many of the boys and girls of our public schools, it may help to advance the great cause of popular education.

EDWARD BROOKS.

STATE NORMAL SCHOOL,
Millersville, Pa., July 4, 1873.

Suggestions to Teachers.

THE attention of teachers is respectfully solicited to the following Methods of Recitation. Some of them are preferable to others, but all may occasionally be used with advantage.

Common Method.—By this method the problems are read by the teacher and assigned promiscuously, the pupils not being permitted to use the book during recitation, nor retain the condition of the problems by means of pencil and paper, as is sometimes done. The pupil selected by the teacher arises, repeats the problem, and gives the solution, at the close of which the mistakes that may have been made should be corrected by the class or teacher.

Silent Method.—By this method the teacher reads a problem to the class, and then the pupils silently solve it, indicating the completion of the solution by the upraised hand. After the whole class, or nearly the whole class, have finished the solution, the teacher calls upon some member, who arises, repeats the problem, and gives the solution, as in the former method.

By this method the whole class must be exercised upon every problem, thus securing more discipline than by the preceding method. It, however, requires more time than the first; hence, not so many problems can be solved at a recitation. We prefer the first method for advanced pupils, and the second, at least a portion of the time, with younger pupils.

Chance Assignment.—This method differs from the first only in the assignment of the problems.—The teacher marks the number of the lesson and the number of the problem upon small pieces of paper, which the pupils may take out of a box passed around by the teacher or some member of the class. The teacher then, after reading a problem, instead of calling upon a pupil, merely gives the number of the problem, the person having the number arising, repeating, and solving it. By this method the teacher is relieved of all responsibility with reference to hard and easy problems, and it is also believed that better attention is secured with it. It is particularly adapted to reviews and public examinations.

Double Assignment.—By this method the pupil who receives the problem from the teacher arises, repeats it, and then assigns it to some one else to solve. It may be combined with either the first or second methods. The objects of this method are variety and interest.

Method by Parts.—By this method different parts of the same problem are solved by different pupils. The teacher reads the problem and assigns it to a pupil, and after he has given a portion of the solution, another is called upon, who takes up the solution at the point where the first stops; the second is succeeded in like manner by a third; and so on, until the solution is completed. The object of this method is to secure the attention of the whole class, which it does very effectually. It is particularly suited to a large class consisting of young pupils.

Unnamed Method.—By this method the teacher reads and assigns several problems to different members of the class, before requiring any solutions, after which those who have received problems are called upon in the order of assignment for their solutions. The advantages of this method

are, first, the pupil, having some time to think of the problem, is enabled to give the solution with more promptness and accuracy, and, secondly, the necessity of retaining the numbers and their relations in the mind for several minutes affords a good discipline to the memory.

CHOOSING SIDES.—This is a modification of the old spelling-class method, and is one calculated to elicit a very great degree of interest. By it two pupils, appointed by the teacher, select the others, thus forming two parties for a trial of skill, as in a game of cricket or base ball. The problems may be assigned alternately to the sides, by the teachers, by chance, by the leaders of the sides, or in any other way that may be agreed upon by the teacher and class.

In regard to these methods, the first, second, and third are probably the best for the usual recitations, but the other methods can very profitably be employed with younger classes, or, in fact, with any class, to relieve monotony and awaken interest. With advanced pupils we prefer the first method, or the first combined with the third.

ERRORS TO BE AVOIDED.

There is a large number of errors to which pupils in every section of the country are liable, a few of which we will mention. We classify them as errors of Position and errors of Expression.

ERRORS OF POSITION.—Pupils are exceedingly liable to assume improper positions and awkward attitudes during recitation, such as leaning on the desk or against the wall, putting the foot upon a seat, jamming the hands into the pockets, particularly when the problem is hard, playing with a button, watch-chain, etc. All of these faults should be carefully guarded against, for reasons so obvious that they need not be mentioned. An erect and graceful carriage, aside from its relation to health, is of advantage to every lady and gentleman.

ERRORS OF EXPRESSION.—Under this head we include errors of Articulation, Pronunciation, Grammar, etc. There is quite a large number of words which pupils in their haste mispronounce, and also quite a large number of combinations, which by a careless enunciation make ridiculous sense or nonsense. We will call attention to a few of them, suggesting to the teacher to correct these and others he may notice.

And is often called *an : for* is called *fur ; of* is pronounced as if the *o* was omitted ; words commencing with *wh*, as *when, which, where*, etc., are pronounced as if spelled *wen, wich, were*, etc.

Gave him is called *gavim ; did he* is called *diddy ; had he* is called *haddy ; give him* is called *givin ; give her* is called *giver ; which is* is often changed into *witches ;* and *how many* is frequently transformed into *hominy*. How many did each earn is often rendered Hominy did *e churn*.

A very common error, and one exceedingly difficult to correct, is involved in the following solution : "If 2 apples cost 6 cents, one apple will cost *the* ½ of 6 cents, which *are* 3 cents." Here *the* is superfluous, and *are* is ungrammatical.

The following is a frequent error : *If one apple cost* 3 *cents, for* 12 *cents you can buy as many apples as* 3 *is contained in* 12, *which are* 4 *times.*" The objections are—first, 3 is not contained any *apples* in 12 ; secondly, the result obtained is *times*, when it should be *apples*, or a *number* which applies to both times and apples. The solution should be, " *You can buy as many apples for* 12 *cents as* 3 *is contained times in* 12, *which are* 4."

NEW

NORMAL MENTAL ARITHMETIC.

SECTION I.

FUNDAMENTAL OPERATIONS.

LESSON I.

Addition.

IF I have 2 cents in one hand, and 1 cent in the other hand, how many cents have I in both hands?

Solution.—If I have 2 cents in one hand, and 1 cent in the other hand, in both hands I have 2 cents and 1 cent, which are 3 cents.

2. If I have 3 cents in one pocket, and 1 cent in another pocket, how many cents have I in both pockets?

3. John has 3 apples, and William has 2 apples; how many apples have they both?

4. Mary has 4 peaches, and Sarah has 2 peaches; how many peaches have they both?

5. In a certain class there are 5 boys and 2 girls; how many pupils are there in the class?

6. There were 6 birds in the garden, when 2 more flew in; how many birds were in the garden then?

7. How many are 3 and 2?

Solution.—3 and 2 are 5.

8. How many are 2 and 2? 5 and 2? 4 and 2? 7 and 2? 9 and 2? 8 and 2?

9. How many are 6 and 2? 9 and 2? 11 and 2? 10 and 2? 12 and 2? 14 and 2?

10. How many are 13 and 2? 15 and 2? 18 and 2? 17 and 2? 16 and 2? 20 and 2?

11. John paid 4 cents for nuts, and 2 cents for candy; what did he pay for both?

12. Harry sold his ball for 12 cents, and his top for 2 cents; how much did he receive for both?

13. Charles caught 13 rabbits in a trap, and Willie caught 2 rabbits; how many did both catch?

14. How many are

2 and 3?	6 and 3?	10 and 3?	17 and 3?
4 and 3?	8 and 3?	13 and 3?	20 and 3?
3 and 3?	9 and 3?	14 and 3?	22 and 3?
5 and 3?	11 and 3?	16 and 3?	23 and 3?
7 and 3?	12 and 3?	18 and 3?	24 and 3?

15. Mary had 13 cents, and her sister had 3 cents; how many cents did they both have?

16. John had 17 books, and William had 3 books; how many books did they together have?

17. How many are

2 and 4?	3 and 4?	11 and 4?	18 and 4?
4 and 4?	5 and 4?	13 and 4?	19 and 4?
6 and 4?	8 and 4?	14 and 4?	21 and 4?
7 and 4?	10 and 4?	16 and 4?	20 and 4?
9 and 4?	12 and 4?	15 and 4?	22 and 4?

18. Mary is 12 years old; how old will she be if she lives 4 years longer?

19. Edward bought 17 marbles, and found 4 marbles; how many marbles had he then?

20. How many are

2 and 5?	5 and 5?	11 and 5?	17 and 5?
4 and 5?	8 and 5?	13 and 5?	19 and 5?
3 and 5?	9 and 5?	15 and 5?	18 and 5?

6 and 5?	10 and 5?	14 and 5?	20 and 5?
7 and 5?	12 and 5?	16 and 5?	21 and 5?

21. Mary had 18 little rabbits, and Nellie had 5 little rabbits; how many did they both have?

22. Peter saw 12 robins in a field, and 5 robins in a tree; how many robins did he see in all?

23. How many are

2 and 6?	7 and 6?	12 and 6?	16 and 6?
3 and 6?	9 and 6?	14 and 6?	19 and 6?
5 and 6?	8 and 6?	15 and 6?	20 and 6?
4 and 6?	10 and 6?	18 and 6?	22 and 6?
6 and 6?	11 and 6?	17 and 6?	21 and 6?

24. There were 9 eggs in one hen's nest, and 6 in another; how many were there in both?

25. Anson, in a game, caught 13 "fly balls," and Dick caught 6; how many did they both catch?

26. How many are

2 and 7?	7 and 7?	11 and 7?	18 and 7?
3 and 7?	9 and 7?	13 and 7?	17 and 7?
5 and 7?	8 and 7?	14 and 7?	19 and 7?
4 and 7?	10 and 7?	16 and 7?	21 and 7?
6 and 7?	12 and 7?	15 and 7?	22 and 7?

27. Dora gave a beggar boy 12 cents, and his sister 7 cents; how much did she give to both?

28. There were 9 robins in a tree, when 7 more flew in; how many robins were in it then?

29. How many are

2 and 8?	7 and 8?	12 and 8?	17 and 8?
3 and 8?	8 and 8?	13 and 8?	18 and 8?
4 and 8?	9 and 8?	14 and 8?	19 and 8?
5 and 8?	10 and 8?	15 and 8?	20 and 8?
6 and 8?	11 and 8?	16 and 8?	21 and 8?

30. A fly has 6 legs, and a spider has 8 legs; how many legs have they both?

31. There are 12 windows in one room, and 8 in another; how many windows in both rooms?

32. How many are

2 and 9?	7 and 9?	12 and 9?	17 and 9?
3 and 9?	8 and 9?	13 and 9?	18 and 9?
4 and 9?	9 and 9?	14 and 9?	19 and 9?
5 and 9?	10 and 9?	15 and 9?	20 and 9?
6 and 9?	11 and 9?	16 and 9?	21 and 9?

33. A boy bought a whip for 10 cents, and a top for 9 cents; what did he pay for both?

34. If a man rides 15 miles, and walks 9 miles, how far will he travel?

LESSON II.

Addition.

FLOY had 5 roses, and Eddie gave her 3 more; how many roses had she then?

Solution.—If Floy had 5 roses, and Eddie gave her 3 more; she then had 5 roses plus 3 roses, which are 8 roses.

2. Evie has 3 apples, and Willie has 4 apples; how many apples have they both?

3. Fanny has 4 peaches, and her sister has 5 peaches; how many peaches have they both?

4. James is 3 years old, and Sarah is 6 years old; what is the sum of their ages?

5. Joseph bought 6 peaches, and his brother gave him 4 peaches; how many peaches had he then?

6. I paid 10 cents for a slate, and 2 cents for a pencil; how many cents did they both cost?

7. A cap cost 3 dollars, and a coat 12 dollars; how much did they both cost?

8. Peter had 8 cents, and found 10 cents; how many cents had he then?

9. Martin had 6 birds, and caught nine more; how many birds had he then?

10. A pig cost 5 dollars, and a sheep cost 7 dollars; what did they both cost?

11. If Cyrus had 9 cows, and bought 11 more, how many would he then have?

12. A saddle cost 8 dollars, and a set of harness 13 dollars; how much did they both cost?

13. Sally had 10 pins in her cushion, and put in 9 more; how many were then in the cushion?

14. If James rode 8 miles, and walked 13 miles, how far did he travel?

15. Mustard had 10 sisters and 11 brothers; how many children are there in the family?

16. Taylor had 5 horses, and his brother sold him 13; how many horses had he then?

17. In a garden there are 9 plum trees, and 11 peach trees; how many trees in the garden?

18. Required the cost of Hunter's coat, if the cloth cost 12 dollars, and the making cost 5 dollars.

19. Rose gave 13 cents to her brother, and 10 to her sister; how many cents did she give away?

20. Mary's mother gave her 11 apples, and her father gave her 7; how many did they both give her?

21. If I have 12 pencils, and find 5 more, how many pencils will I then have?

22. Russell gave 3 cents for a top, 4 for a whip, and 5 for a book; what did they all cost?

23. Louisa had 5 peaches, her mother gave her 6, and her sister gave her 7; how many had she then?

24. Ruth bought 5 yards of silk for a cloak, and 21 yards for a dress; how many yards did she buy?

25. The head of a fish is 6 inches long, the tail 8, and the body 10; what is the length of the fish?

26. Philo caught 12 fish, Milo 9, and Nero 6; how many did they all catch?

27. Howard shot 11 robins, Howell 12, and Hoyt 13; how many did they all shoot?

28. A man bought a robin for 10 cents, a jay for 20 cents, and a wren for 30 cents; required their cost.

29. A miller bought some rye for 25 dollars, and some wheat for 26 dollars; what did he pay for both?

30. Addie has 9 roses, Lizzie has 15, and Amy has 20; how many have they all?

31. A lady gave 6 cents for needles, 12 cents for thread, and 10 cents for muslin; what was the cost of all?

32. A merchant sold some rice for 15 dollars, some sugar for 17 dollars, and some molasses for 22 dollars; required the amount received.

33. How many are 4 and 8? 5 and 7? 3 and 9? 5 and 12? 8 and 17? 6 and 23? 7 and 15? 8 and 26? 10 and 28? 10 and 32? 12 and 20? 17 and 15?

Remark.—The sum of 4 and 8 is the same as 8 and 4; and 8 and 4 are 12.

34. How many are 5 and 21? 8 and 23? 9 and 31? 10 and 16? 3 and 19? 8 and 27? 10 and 27? 11 and 11? 12 and 12? 12 and 24?

35. How many are 6 and 16? 7 and 17? 8 and 18? 9 and 19? 10 and 20? 11 and 21? 12 and 22? 13 and 23? 20 and 30? 11 and 22?

36. How many are 2 and 12? 3 and 13? 4 and 14? 5 and 15? 6 and 26? 7 and 27? 8 and 28? 9 and 29? 10 and 30? 11 and 31?

37. How many are 2 and 17? 3 and 27? 4 and 36? 5 and 46? 12 and 41? 12 and 29? 12 and 37? 13 and 57? 18 and 65? 56 and 44?

The process of finding the *sum* of two or more numbers is called *addition.*

The symbol + is the sign of addition. It is read *plus*, and when placed between two numbers denotes that they are to be added together.

The symbol = is the sign of *equality*, and is read *equals*, or *is equal to.* Thus, 6 + 4 = 10 is read "6 *plus* 4 *equals* 10."

LESSON III.

Subtraction.

IF I have 4 cents, and give away 2 cents, how many cents shall I have remaining?

Solution.—If I have 4 cents, and give away 2 cents, I shall have remaining the difference between 4 cents and 2 cents, which is 2 cents.

2. If I have 5 apples, and give 2 of them away, how many apples shall I have remaining?

3. Sarah had 6 roses, and gave 2 of them to Jane; how many roses had she left?

4. How many are 2 from 5? 2 from 7? 2 from 9? 2 from 12? 2 from 11? 2 from 13? 2 from 15?

5. How many are 2 from 10? 2 from 14? 2 from 16? 2 from 19? 2 from 18? 2 from 17? 2 from 21?

6. How many are

3 from 5? 3 from 8? 3 from 14? 3 from 20?
3 from 7? 3 from 10? 3 from 12? 3 from 21?
3 from 9? 3 from 13? 3 from 15? 3 from 22?
3 from 6? 3 from 11? 3 from 16? 3 from 24?

7. There were 12 birds in a tree, and 3 of them flew away; how many birds remained?

8. There were 20 cows in a field, and 3 of them jumped out; how many cows remained in the field?

9. How many are

4 from 7? 4 from 10? 4 from 15? 4 from 19?
4 from 9? 4 from 11? 4 from 16? 4 from 22?
4 from 8? 4 from 13? 4 from 18? 4 from 21?
4 from 12? 4 from 14? 4 from 20? 4 from 24?

10. 4 pupils were absent from a class of 17 pupils; how many pupils were in the class?

11. Ella bought 20 needles, and gave Jennie 4 of them; how many needles did Ella retain?

12. How many are

5 from 7? 5 from 11? 5 from 19? 5 from 17?
5 from 8? 5 from 13? 5 from 18? 5 from 21?
5 from 10? 5 from 14? 5 from 20? 5 from 23?
5 from 12? 5 from 15? 5 from 22? 5 from 24?

13. If I have 20 books, and sell 5 of them, how many books shall I have left?

14. A boy found 24 eggs in the barn, and broke 5 of them; how many of them remained unbroken?

15. How many are

6 from 8? 6 from 11? 6 from 16? 6 from 22?
6 from 10? 6 from 13? 6 from 19? 6 from 24?
6 from 9? 6 from 15? 6 from 18? 6 from 25?
6 from 12? 6 from 14? 6 from 20? 6 from 26?

16. How many birds will escape if I shoot 6 birds out of a flock of 23 birds?

17. If Hannah has 17 cents, and Hanson has 6 cents, how many cents has Hannah more than Hanson?

18. How many are

7 from 10? 7 from 11? 7 from 16? 7 from 24?
7 from 9? 7 from 15? 7 from 19? 7 from 26?
7 from 12? 7 from 17? 7 from 20? 7 from 27?
7 from 14? 7 from 13? 7 from 18? 7 from 28?

19. If Harold earns 23 dollars a month, and spends 7 dollars, how much money will he save?

20. A man's salary was 25 hundred dollars a year, and he spent 7 hundred dollars; how much remained?

21. How many are

8 from 11? 8 from 13? 8 from 20? 8 from 23?
8 from 10? 8 from 15? 8 from 17? 8 from 22?
8 from 12? 8 from 14? 8 from 19? 8 from 24?
8 from 16? 8 from 18? 8 from 21? 8 from 25?

22. If a drover has 20 sheep, and sells 8 sheep, how many sheep remain?

23. A hunter saw 24 rabbits, and shot 8 of them; how many escaped?

24. How many are

9 from 12?	9 from 16?	9 from 21?	9 from 30?
9 from 14?	9 from 18?	9 from 23?	9 from 31?
9 from 11?	9 from 20?	9 from 22?	9 from 32?
9 from 13?	9 from 17?	9 from 24?	9 from 40?
9 from 15?	9 from 19?	9 from 25?	9 from 45?

25. If there are 20 crows on a tree, and 9 fly away, how many crows will then be left?

26. Sophia received 23 words, and missed 9 of them; how many did she spell correctly?

27. Begin at 2 and count to 50 by 2's. Begin at 50 and count by 2's backward to 2.

28. Begin at 3 and count to 30 and back again by 3's; begin at 2 and count to 29 and back by 3's.

29. Count by 4's from 4 to 48 and back; from 3 to 43 and back; from 2 to 42 and back; from 1 to 41 and back.

30. Count by 5's from 5 to 50 and back; from 4 to 54 and back; from 3 to 53 and back; from 2 to 52 and back; from 1 to 51 and back.

LESSON IV.

Subtraction.

A MAN bought 10 sheep, and sold 4 of them; how many sheep had he remaining?

Solution.—If a man bought 10 sheep, and sold 4 of them, he had remaining 10 sheep minus 4 sheep, which are 6 sheep.

2. Hunter had 6 apples, and gave 2 of them away; how many had he remaining?

3. Morgan, having 9 peaches, gave his sister 3 of them; how many had he left?

4. Ada culled 10 roses, and gave Lydia 5 of them; how many did Ada retain?

5. A man bought 12 lemons, and sold 7 of them; how many lemons remained unsold?

6. Reuben, finding 13 cents, spent 8 of them; how many cents had he remaining?

7. A watch was bought for 20 dollars, and sold for 27 dollars; what was the gain?

8. Mary has 28 pins, and Susan 15; how many more has Mary than Susan?

9. A cow was bought for 27 dollars, and sold for 10 dollars; required the loss.

10. In a school of 40 pupils only 20 are present; how many are absent?

11. A house cost 8 thousand dollars, and was sold for 12 thousand dollars; required the gain.

12. William said he found 43 marbles, and lost 13 of them; how many remained?

13. A farmer, having 27 cows, sold 17 of them; how many cows had he remaining?

14. How many are 10 from 15? 10 from 18? 10 from 16? 10 from 21? 10 from 17? 10 from 23?

15. How many are 10 from 19? 10 from 20? 10 from 22? 10 from 26? 10 from 28? 10 from 30?

16. How many are 11 from 11? 11 from 13? 11 from 12? 11 from 15? 11 from 14? 11 from 16?

17. How many are 11 from 17? 11 from 19? 11 from 21? 11 from 23? 11 from 25? 11 from 24?

18. How many are 12 from 15? 12 from 13? 12 from 16? 12 from 14? 12 from 18? 12 from 20?

19. How many are 12 from 17? 12 from 19? 12 from 22? 12 from 23? 12 from 26? 12 from 28?

20. How many are 9 less 5? 11 less 7? 13 less 9? 15 less 11? 18 less 12?

21. How many are 16 less 8? 14 less 6? 12 less 4? 10 less 2? 14 less 8?

22. Required the value of 12 minus 8; 13 minus 7; 14 minus 6; 15 minus 5; 16 minus 4.

23. Required the value of 10 minus 5; 12 minus 6; 14 minus 7; 16 minus 8; 18 minus 9.

24. Required the value of 11 minus 7; 14 minus 8; 17 minus 9; 20 minus 10; 23 minus 11.

25. Required the value of 6 minus 2; 10 minus 4; 14 minus 6; 8 minus 6; 22 minus 10; 26 minus 12.

26. Count by 6's from 6 to 42 and back; from 5 to 41 and back; from 4 to 40 and back; from 3 to 45 and back; from 2 to 44 and back, etc.

27. Count by 7's from 7 to 56 and back; from 6 to 48 and back; from 5 to 47 and back; from 4 to 46 and back; from 3 to 45 and back, etc.

28. Take the number 4, add 6, subtract 5, add 7, subtract 6, add 8, subtract 7, add 9, subtract 8, and name the result.

29. Take the number 10, add 3, subtract 2, add 4, subtract 3, add 5, subtract 4, add 6, subtract 5, add 7, subtract 6, and name the result.

30. Take the number 12, add 2, subtract 3, add 3, subtract 4, add 4, subtract 5, add 5, subtract 6, add 6, subtract 7, and name the result.

31. Take the number 10, add 10 and subtract 5, add 5 and subtract 10, add 12 and subtract 6, add 6 and subtract 12, add 20 and subtract 10, add 10 and subtract 20, and name the result.

The process of finding the *difference* between two numbers is called *subtraction*. The larger number is called the *minuend*, the smaller the *subtrahend*, the result the *difference* or *remainder*.

The symbol — is the sign of subtraction; it indicates that the number at the right is to be subtracted from the number at the left. It is read *minus;* thus $12 - 7 = 5$ is read "12 *minus* 7 *equals* 5."

NOTE.—The teacher may give other similar exercises, counting forward and backward, to any extent desired.

LESSON V.

Addition and Subtraction.

A MAN had 6 cows, and bought 7 cows, and sold 8 cows; how many cows had he then?

Solution.—If a man had 6 cows and bought 7 cows, he then had 6 cows plus 7 cows, which are 13 cows; if he then sold 8 cows, he had 13 cows minus 8 cows, which are 5 cows.

2. Silas, having 3 dollars, found 6, and then lost 4; how many had he remaining?

3. A boy, having 12 apples, bought 6 more, and then sold 8; how many had he left?

4. A man sold a colt for 25 dollars, which is 3 dollars more than the colt cost; required the cost.

5. John, having 20 peaches, ate 4, and gave his sister 6; how many had he remaining?

6. Philo, having 12 cents, spent 7, and then found 9; how many had he then?

7. What is the value of $6+8-4$? $7+5-6$? $5+10-12$?

What is the value

8. Of $7+6-5$?
9. Of $3+8-9$?
10. Of $5+6-7$?
11. Of $7+8-9$?
12. Of $9-6+3$?
13. Of $8-5+10$?
14. Of $6-3+12$?
15. Of $10-6+4$?
16. Of $16-7+8$?
17. Of $18-6+3$?
18. Of $15-8+2$?
19. Of $16-7+8$?
20. Of $17-9+4$?
21. Of $22-12+13$?

22. Paxton, having 16 peaches, gave 6 to James, and 7 to Henry; how many had he remaining?

23. Edwin had 24 cents, and found 6 cents, and then spent 11 cents; how many cents had he remaining?

24. A man sold 13 cows, then bought 10, and then had 12; how many had he at first?

25. A merchant bought goods to the amount of 27 dollars; how must he sell them to gain 11 dollars?

26. A man, having a certain number of cows, bought 6 and sold 10, and then had none left; how many had he at first?

27. Morris, having a certain number of books, bought 10, and giving 30 to his sister, had none remaining; how many books had he at first?

28. Two boys commenced playing with 20 marbles each; at the close of the game one had 16; how many had the other?

29. Thomas had 11 cents, Susan gave him 12, and Walton gave him enough to make his number 30; how many did Walton give him?

30. Mr. A gave 35 dollars for a case of goods, and paid 4 dollars for cartage; how must he sell them to gain 11 dollars?

31. How many are 4 plus 6 minus 5? 7 plus 8 minus 10? 8 plus 12 minus 9? 12 plus 13 minus 14? 16 plus 20 minus 30? 7 plus 15 minus 12?

32. How many are 7 and 20 minus 13? 9 and 13 minus 16? 13 and 15 minus 16? 18 and 19 minus 20? 16 and 17 minus 18? 20 and 30 minus 40?

33. How many are 6 plus 8 minus 10? 7 plus 9 minus 11? 8 plus 10 minus 12? 9 plus 11 minus 13? 10 plus 12 minus 14? 15 plus 17 minus 19?

34. How many are 6 plus 10 minus 14? 7 plus 11 minus 15? 8 plus 12 minus 16? 9 plus 13 minus 17? 10 plus 14 minus 18? 11 plus 15 minus 19?

35. How many are 4 plus 24 minus 14? 6 plus 26 minus 16? 7 plus 27 minus 17? 5 plus 25 minus 15? 8 plus 28 minus 18? 9 plus 29 minus 19?

36. How many are 7 plus 37 minus 27? 10 plus 30 minus 20? 11 plus 31 minus 21? 12 plus 32 minus 22? 13 plus 33 minus 23? 14 plus 34 minus 24?

37. How many are 4 and 4 minus 2 and 2? 6 and 6 minus 3 and 3? 12 and 12 minus 6 and 6? 23 and 23 minus 13 and 13? 27 and 27 minus 17 and 17?

LESSON VI.

Multiplication.

IF one apple costs 3 cents, how many times 3 cents will 2 apples cost?

2. How many are 2 times 3 cents?

Solution.—Two times 3 cents are 3 cents plus 3 cents, or 6 cents; hence, 2 times 3 cents are 6 cents.

NOTE.—Pupils who are familiar with the multiplication-table will name the products without performing the addition. Those who are not will *make their own multiplication-table* by addition.

3. How many are 2 times 1? 2 times 2? 2 times 3? 2 times 4? 2 times 5? 2 times 6?

4. How many are 2 times 7? 2 times 8? 2 times 9? 2 times 10? 2 times 11? 2 times 12?

5. If 1 melon is worth 3 cents, how much are 2 melons worth, at the same rate?

Solution.—If 1 melon is worth 3 cents, 2 melons are worth 2 times 3 cents, which are 6 cents.

6. If 1 pair of boots is worth 8 dollars, what is the value of 2 pairs of boots, at the same rate?

7. How many are

3 times 1? 3 times 4? 3 times 7? 3 times 10?
3 times 2? 3 times 5? 3 times 8? 3 times 11?
3 times 3? 3 times 6? 3 times 9? 3 times 12?

8. If 1 orange is worth 4 cents, how much are 3 oranges worth, at the same rate?

9. At the rate of 6 dollars a barrel, what will 3 barrels of flour cost?

10. How many are

4 times 1? 4 times 4? 4 times 7? 4 times 10?
4 times 2? 4 times 5? 4 times 8? 4 times 11?
4 times 3? 4 times 6? 4 times 9? 4 times 12?

11. At 6 cents a yard, what will 4 yards of ribbon cost?

12. If Susan writes 9 lines a day in her copy-book, how many lines will she write in 4 days?

13. How many are

5 times 1? 5 times 4? 5 times 7? 5 times 10?
5 times 2? 5 times 5? 5 times 8? 5 times 11?
5 times 3? 5 times 6? 5 times 9? 5 times 12?

14. How many boys are seated on 5 benches, if there are 8 boys sitting on each bench?

15. If a boy walks 11 miles each day for 5 days, how far will he walk?

16. How many are

6 times 1? 6 times 4? 6 times 7? 6 times 10?
6 times 2? 6 times 5? 6 times 8? 6 times 11?
6 times 3? 6 times 6? 6 times 9? 6 times 12?

17. How much will 6 barrels of fish cost, at the rate of 8 dollars a barrel?

18. How much will 6 pounds of candles cost, at the rate of 12 cents a pound?

19. How many are

7 times 1? 7 times 4? 7 times 7? 7 times 10?
7 times 2? 7 times 5? 7 times 8? 7 times 11?
7 times 3? 7 times 6? 7 times 9? 7 times 12?

20. What will 7 bunches of roses cost, at the rate of 6 cents a bunch?

21. If it takes 12 steps to cross a room once, how many steps will it take to cross it 7 times?

22. How many are

8 times 1? 8 times 4? 8 times 7? 8 times 10?
8 times 2? 8 times 5? 8 times 8? 8 times 11?
8 times 3? 8 times 6? 8 times 9? 8 times 12?

23. At the rate of 9 dollars a week, how much can I earn in 8 weeks?

24. If a stage-coach goes 6 miles in 1 hour, how far can it go in 8 hours?

25. How many are

9 times 1? 9 times 4? 9 times 7? 9 times 10?
9 times 2? 9 times 5? 9 times 8? 9 times 11?
9 times 3? 9 times 6? 9 times 9? 9 times 12?

26. How much must I pay for 9 primers, at the rate of 7 cents apiece?

27. What will 9 pounds of sturgeon cost, at the rate of 12 cents a pound?

28. How many are

10 times 1? 10 times 4? 10 times 7? 10 times 10?
10 times 2? 10 times 5? 10 times 8? 10 times 11?
10 times 3? 10 times 6? 10 times 9? 10 times 12?

29. In a school there are 5 boys on each of 10 seats; how many boys in the school?

30. An orchard has 10 rows of trees with 9 trees in each row; how many trees in the orchard?

31. How many are

11 times 1? 11 times 4? 11 times 7? 11 times 10?
11 times 2? 11 times 5? 11 times 8? 11 times 11?
11 times 3? 11 times 6? 11 times 9? 11 times 12?

32. How many panes of glass in 11 windows, if there are 12 panes in each window?

33. If a cow gives 9 quarts of milk in a day, how many quarts will she give in 11 days?

34. How many are

12 times 1? 12 times 4? 12 times 7? 12 times 10?
12 times 2? 12 times 5? 12 times 8? 12 times 11?
12 times 3? 12 times 6? 12 times 9? 12 times 12?

35. A stage-coach runs 9 miles an hour; how far will it run at this rate in 12 hours?

36. If a coal-train runs at the rate of 12 miles an hour, how far will it run in 12 hours?

LESSON VII.

Multiplication.

WHAT will 4 apples cost, at 2 cents apiece?

Solution.—If 1 apple costs 2 cents, 4 apples will cost 4 times 2 cents, which are 8 cents.

2. What will 5 pears cost, at 3 cents apiece?

3. What will 4 melons cost, at 5 cents apiece?

4. What will 8 sheep cost, at 6 dollars apiece?

5. What will 12 turkeys cost, at 3 dimes apiece?

6. At 20 dollars apiece, what will 7 cows cost?

7. At 30 dollars apiece, what will 12 pictures cost?

8. At 20 cents apiece, what will 12 arithmetics cost?

9. At 40 cents apiece, what will 9 ducks cost?

10. How far will a man travel in 9 days, at the rate of 12 miles a day?

11. A boy has 12 apples, and 5 times as many peaches; required the number of peaches.

12. Mary has 11 apples, and John has 7 times as many pears; how many pears has John?

13. A has 10 dollars, and B has 9 times as many; how much money have they both?

14. A farmer sold 4 horses, and then bought 6 times as many; how many did he buy?

15. How many dimes must be paid for 9 books, at the rate of 14 dimes each?

16. Amos saw 12 flocks of pigeons, with 20 pigeons in each flock? how many pigeons did he see?

17. Mary found 10 pins, and then bought 8 times as many as she found; how many had she then?

18. How much will 7 yards of muslin cost, at 8 cents a yard?

19. What is the cost of 11 yards of ribbon, at the rate of 11 cents a yard?

20. If 3 men can do a piece of work in 5 days, how long will it take 1 man to do it?

21. If 5 men can do a piece of work in 20 days, how long will it require 1 man to do it?

22. How far will a man travel in 9 hours, at the rate of 3 miles an hour?

23. If 9 men mow a field of grass in 14 days, how long will it take 1 man to mow it?

24. If 1 melon is worth 3 peaches, how many peaches are 7 melons worth?

25. In an orchard there are 11 rows of trees, and 20 trees in each row; how many trees are there in the orchard?

26. A boy borrowed 6 cents from a friend, and then earned 7 times as much as he borrowed; how many cents had he then?

27. A merchant, having 10 melons, sold 6, and then bought 5 times as many as he sold; how many had he then?

28. B, having 20 sheep, sold 12, and then bought 4 times as many as remained; how many did he then have?

29. Mason earned 10 dollars a week, and paid 3 dollars a week for his board; how much could he save in 5 weeks?

30. Passmore earned 3 dollars a day, and paid 1 dollar a day for his board; how much could he save in a week?

31. Morton earned 30 dollars a month; he paid 3 dollars a week for board, and 1 dollar a week for other expenses; how much could he save in a year?

32. Thomas travels 5 miles an hour, and John travels 3 miles an hour; how much farther will Thomas travel in 12 hours than John?

33. How many are 6 times 6, plus 6? 5 times 8, plus 8? 7 times 10, plus 10? 4 times 11, plus 11? 8 times 4, plus 4? 5 times 9, plus 9?

34. How many are 3 times 4, plus 5? 6 times 7, plus

8? 8 times 9, plus 10? 9 times 10, plus 11? 10 times 11, plus 12? 11 times 12, plus 13?

35. How many are 4 times 4, minus 5? 5 times 7, minus 6? 7 times 9, minus 8? 8 times 10, minus 9? 9 times 12, minus 11? 11 times 12, minus 13?

36. How many are 7 and 7 times 8? 5 and 5 times 6? 8 and 8 times 4? 6 and 6 times 7? 9 and 9 times 11? 10 and 10 times 12?

37. How many are 4 and 5 and 4 times 5? 5 and 6 and 5 times 6? 6 and 7 and 6 times 7? 7 and 8 and 7 times 8? 8 and 9 and 8 times 9?

38. How many are 23 minus 2 times 3? 34 minus 3 times 4? 25 minus 2 times 5? 37 minus 3 times 7? 49 minus 4 times 9?

39. B and C start from the same place, and travel in opposite directions, B at the rate of 5 and C at the rate of 4 miles an hour; how far are they apart in 6 hours?

40. Two men start at the same place, and travel in the same direction, the one 7 and the other 5 miles an hour; how far will they be apart in 10 hours?

The process of taking one number as many times as there are units in another is called *multiplication.* The number multiplied is the *multiplicand;* the number by which we multiply is the *multiplier;* the result is the *product.*

The symbol $\times$ is the sign of multiplication; when placed between two numbers, it denotes that one is to be multiplied by the other.

LESSON VIII.

Division.

1. 6 are how many times 2?

Solution.—6 are 3 times 2, since 3 times 2 are 6.

2. How many times 2 are 4? 8? 10? 12? 14?

3. How many times 3 are 6? 9? 15? 12? 18?

4. How many times 4 are 8? 16? 12? 20? 28?

5. How many times 5 are 10? 20? 15? 25? 35?

6. How many 2's are there in 6?

Solution.—There are 3 *twos* in 6, since 3 times 2 are 6.

7. How many 6's in 18? in 30? in 12? in 24?

8. How many 7's in 14? in 21? in 35? in 42?

9. How many 8's in 16? in 32? in 24? in 48?

10. How many 9's in 18? in 27? in 54? in 45?

11. How many times is 3 contained in 12?

Solution.—3 is contained in 12, 4 times, since 4 times 3 are 12.

12. How many times is 2 contained

in 2?	in 8?	in 16?	in 20?
in 4?	in 12?	in 14?	in 22?
in 6?	in 10?	in 18?	in 24?

13. If 1 apple costs 2 cents, how many apples can I buy for 12 cents?

Solution.—If 1 apple costs 2 cents, for 12 cents I can buy as many apples as 2 cents are contained times in 12 cents, which are 6.

14. If I can buy 1 peach for 2 cents, how many peaches can I buy for 18 cents?

15. How many times is 3 contained

in 3?	in 12?	in 21?	in 27?
in 9?	in 18?	in 24?	in 33?
in 6?	in 15?	in 30?	in 36?

16. If I pay 3 dollars a yard, how many yards of silk can I buy for 21 dollars?

17. If 1 step is 3 feet long, how many steps will I take in going 36 feet?

18. How many times is 4 contained

in 4?	in 16?	in 32?	in 40?
in 12?	in 24?	in 28?	in 48?
in 8?	in 20?	in 36?	in 44?

19. If a squirrel gathers 4 chestnuts a day, how long will it take him to gather 24 chestnuts?

20. Mary has 36 spools; how many piles will they make if she puts 4 spools in each pile?

21. How many times is 5 contained

in 5?	in 15?	in 30?	in 55?
in 10?	in 25?	in 40?	in 50?
in 20?	in 35?	in 45?	in 60?

22. If Edward goes to school 5 days each week, in how many weeks will he attend 30 days?

23. There are 55 boys seated in a school, and 5 boys on each seat; how many seats in the school?

24. How many times is 6 contained

in 6?	in 18?	in 42?	in 66?
in 12?	in 36?	in 48?	in 54?
in 24?	in 30?	in 60?	in 72?

25. Susan wrote 42 lines by writing 6 lines a day; how many days was she writing them?

26. Adam walked 72 miles by walking 6 miles a day; how long did it take him?

27. How many times is 7 contained

in 7?	in 28?	in 42?	in 77?
in 21?	in 35?	in 56?	in 70?
in 14?	in 49?	in 63?	in 84?

28. There are 7 days in 1 week; how many weeks are there in 56 days?

29. How many settees will seat 70 boys, if each settee seats 7 boys?

30. How many times is 8 contained

in 8?	in 24?	in 48?	in 88?
in 16?	in 40?	in 64?	in 80?
in 32?	in 56?	in 72?	in 96?

31. A railroad-car has 8 wheels; how many cars will 48 car-wheels supply?

32. At 8 cents apiece, how many slates can be bought for 88 cents?

33. How many times is 9 contained

in 9?	in 63?	in 45?	in 99?
in 18?	in 36?	in 72?	in 90?
in 27?	in 54?	in 81?	in 108?

34. A boy has 90 marbles; how many groups of 9 marbles can he make out of them?

35. If 9 musk-melons fill a basket, how many baskets will 108 such melons fill?

36. How many times is 10 contained

in 10?	in 50?	in 70?	in 100?
in 20?	in 40?	in 90?	in 110?
in 30?	in 60?	in 80?	in 120?

37. How many yards of muslin can I buy for 70 cents, at the rate of 10 cents a yard?

38. A farmer expended 20 dollars for sheep, at the rate of 10 dollars each; how many did he buy?

39. How many times is 11 contained

in 11?	in 44?	in 55?	in 121?
in 33?	in 66?	in 88?	in 110?
in 22?	in 77?	in 99?	in 132?

40. If a steamer sails 11 miles an hour, how long will it be in sailing 55 miles?

41. There are 110 trees in an orchard, with 11 trees in each row; how many rows are there?

42. How many times is 12 contained

in 12?	in 48?	in 84?	in 120?
in 24?	in 60?	in 96?	in 132?
in 36?	in 72?	in 108?	in 144?

43. There are 12 months in 1 year; how many years are there in 120 months?

44. There are 132 books on the shelves of a library; how many shelves are there if there are 12 books on each shelf?

LESSON IX.

Division.

AT 3 cents each, how many melons can I buy for 12 cents?

Solution.—If for 3 cents I can buy 1 melon, for 12 cents I can buy as many melons as 3 is contained times in 12, which are 4. Therefore, etc.

2. At 2 dimes apiece, how many books can I buy for 8 dimes?

3. At 4 cents a yard, how many yards of ribbon can I buy for 16 cents?

4. How many apples can I buy for 21 cents, at 3 cents apiece?

5. How many yards of ribbon, at 6 cents a yard, can be bought for 42 cents?

6. A man gave 50 dollars for sheep, at the rate of 5 dollars a head; how many did he buy?

7. If a man travels 4 miles an hour, how long will it take him to travel 48 miles?

8. A man gave 56 cents to some boys, giving them 7 cents each; how many boys were there?

9. How many apples can I buy for 24 cents, at the rate of 8 cents each?

10. A farmer received 88 dollars for sheep, at the rate of 8 dollars each; how many did he sell?

11. How many kegs, of 9 gallons each, can be filled from a hogshead containing 63 gallons of vinegar?

12. How many days must a man work to earn 44 dollars, at the rate of 4 dollars a day?

13. How many melons, at 7 cents apiece, may be bought for 84 cents?

14. How many oranges, at 3 cents each, may be had for 5 lemons, worth 6 cents each?

15. How many yards of lace, at 6 cents a yard, may be bought for 3 yards of muslin, at 12 cents a yard?

16. How many pounds of meat, at 6 cents a pound, will cost as much as 9 yards of ribbon, at 8 cents a yard?

17. If a man digs 10 yards of ditch for 8 dimes a yard, how many bushels of rye, at 4 dimes a bushel, will pay him?

18. How many boxes of wafers, worth 6 cents a box, may be bought for 12 sheets of paper, worth 2 cents a sheet?

19. How much wheat, at 6 dimes a bushel, may be purchased for 12 bushels of corn, worth 5 dimes a bushel?

20. How many knives, at 3 dimes each, can I buy for 2 dimes in money and 7 boxes of raisins, at 4 dimes each?

21. If flour is worth 8 dollars a barrel, how many barrels can be bought for 3 dollars, and 7 barrels of fish, at 11 dollars per barrel?

22. A man gave 9 pencils, worth 5 cents each, for 3 packages of envelopes, worth 11 cents each; what did he lose?

23. How many lamps, at 7 dimes each, can be bought for 9 dimes in money and 6 dozen eggs, at 2 dimes per dozen?

24. How many are 12 plus 6 divided by 6? 25 plus 5 divided by 5? 32 plus 8 divided by 8? 42 plus 7 divided by 7? 45 plus 9 divided by 9?

25. How many are 16 and 4 divided by 4? 21 and 7 divided by 7? 44 and 11 divided by 11? 60 and 12 divided by 12? 96 and 8 divided by 8?

26. How many are 90 minus 9 divided by 9? 80 minus 8 divided by 8? 100 minus 4 divided by 4? 110 minus 10 divided by 10? 144 minus 12 divided by 12?

27. How many are 6 times 8 divided by 4? 7 times 9 divided by 3? 10 times 6 divided by 12? 9 times 9 divided by 3? 12 times 10 divided by 5?

28. How many are 2 times 22 divided by 11? 4 times

14 divided by 7? 3 times 15 divided by 5? 6 times 16 divided by 8? 8 times 18 divided by 12?

29. Twice a number, + 3 times the number, — 4 times the number, + 2 times the number, equals how many times the number?

30. Three times a number, — 2 times the number, + 4 times the number, — 5 times the number, equals how many times the number?

31. Four times Susan's age, + 3 times her age, — 5 times her age, + 2 times her age, — 3 times her age, equals 13 years; how old is Susan?

32. Eight times a number, divided by 4, multiplied by 6, divided by 3, multiplied by 5, divided by 10, equals how many times the number?

33. Five times a number, multiplied by 4, divided by 10, multiplied by 6, divided by 4, multiplied by 2, equals how many times the number?

34. Three times Mary's age, multiplied by 6, divided by 9, multiplied by 4, divided by 8, equals 16 years; how old is Mary?

35. Think of any number, multiply it by 6, divide by 3, multiply by 2, divide by 4, add 10, subtract the number thought of, divide by 5, and the quotient is what?

36. Think of a number, multiply it by 10, divide it by 5, multiply by 3, divide by 6, add 16, subtract the number thought of, divide by 8, and the quotient is what?

The process of finding how many times one number is contained in another is called *division*.

The number to be divided is called the *dividend;* the number we divide by is called the *divisor;* the result of the division is called the *quotient*.

The symbol ÷ is the sign of division; when placed between two numbers, it denotes that the one on the left is to be divided by the one on the right.

LESSON X.

Larger Numbers.

In the previous lessons, numbers, though expressed by figures, have been used without any reference to the principle of units, tens, etc. Pupils have learned to add, subtract, multiply, and divide, without separating the numbers into their elements.

They are now prepared to understand the elements of the Arabic system of notation, which will be explained in this lesson. In some of the problems which follow, and especially toward the latter part of the book, large numbers are used which require these principles.

THE characters which are used to express numbers are called *figures.*

2. When a figure stands by itself, or at the right of other figures, it expresses *ones* or *units.*

3. When a figure stands in the second place toward the left, it expresses *tens.*

4. Analyze the number 36.

Solution.—In 36 the 6 expresses 6 *units,* and the 3 expresses 3 *tens,* and the number is 3 tens and 6 units, or *thirty-six.*

5. Analyze 28, 37, 45, 48, 56, 62, 67, 75, 86, 89, 93, 98.

6. When a figure stands in the third place, it expresses *hundreds;* thus in 436, the 4 expresses 4 *hundreds.*

7. Analyze 235, 268, 354, 375, 483, 789, 846, 562, 784, 804, 906, 574, 875.

8. When a figure stands in the fourth place, it expresses *thousands;* thus in 4568, the 4 expresses 4 *thousands.*

9. Analyze 2372, 5168, 7249, 5873, 6428, 2051, 7504, 8306, 6048, 7008.

10. What is the sum of 20 and 30?

Solution.—20 equals 2 *tens,* and 30 equals 3 *tens,* and 2 *tens* plus 3 *tens* are 5 *tens,* or 50.

11. What is the sum of 30 and 40? 20 and 50? 30 and 60? 40 and 50? 50 and 70? 60 and 80?

12. What is the difference of 40 and 20? 50 and 30? 60 and 20? 70 and 50? 60 and 40? 90 and 40?

13. What is the sum of 28 and 37?

Solution.—8 units and 7 units are 15 units, or 1 ten and 5 units; 2 tens and 3 tens are 5 tens, which, added to 1 ten and 5 units, equals 6 tens and 5 units, or 65.

Solution 2d.—This problem may also be solved thus: 28 and 30 are 58 and 7 are 65.

14. What is the sum of 25 and 38? 27 and 36? 45 and 28? 47 and 65? 67 and 48? 73 and 48?

15. What is the difference between 75 and 32? 86 and 54? 67 and 31? 85 and 43? 78 and 28? 96 and 35?

16. A watch cost 78 dollars, and was sold at a gain of 16 dollars; for what was it sold?

17. I gave 75 dollars for a colt, 46 dollars for a cow, and 23 dollars for a hog; what did I give for all?

18. Subtract 28 from 72.

Solution.—20 from 72 leaves 52, and 8 from 52 leaves 44.

19. Subtract 25 from 43; 35 from 52; 37 from 65; 34 from 60; 45 from 70; 52 from 80; 64 from 90.

20. A man had 72 acres of land, and sold 25 acres; how much remained?

21. If you owed 85 dollars, and paid 36 dollars, how much would you still owe?

22. What is the sum of 123 and 234? of 328 and 246? of 546 and 327? of 486 and 258?

23. Subtract 123 from 138; 134 from 167; 123 from 248; 234 from 467.

24. A bought a horse for 250 dollars, and sold him at a gain of 45 dollars; for what did he sell him?

25. A bought a horse for 175 dollars, paid 25 dollars for his keeping, and sold him for 250 dollars; what was the gain?

26. Multiply 24 by 4.

Solution.—4 times 4 *units* are 16 *units*, which equals 1 *ten* and 6 *units*; 4 times 2 *tens* are 8 *tens*, which, added to the 1 *ten* and 6 *units*, equals 9 *tens* and 6 *units*, or 96.

27. Multiply 18 by 4; 24 by 3; 16 by 5; 15 by 6; 25 by 5; 42 by 8; 56 by 7.

28. If a ton of hay costs 25 dollars, what will 8 tons cost, at the same rate?

29. If a train runs 34 miles an hour, how far will it run in 6 hours?

30. Divide 68 by 2.

Solution.—2 is contained in 6 tens 3 tens times; 2 is contained in 8 units 4 units times; hence the quotient is 3 tens and 4 units, or 34.

31. Divide 68 by 2; 69 by 3; 48 by 4; 55 by 5; 66 by 6; 84 by 4; 96 by 3; 468 by 2.

32. Divide 56 by 2.

Solution.—2 is contained in 5 tens 2 tens times and 1 ten remaining; 1 ten and 6 units equal 16 units; 2 is contained in 16 units 8 units times; hence 56 divided by 2 is 28.

33. Divide 38 by 2; 54 by 2; 76 by 2; 48 by 3; 57 by 3; 84 by 3; 72 by 3.

34. Divide 64 by 4; 76 by 4; 96 by 4; 104 by 4; 112 by 4; 136 by 4; 216 by 4.

35. Multiply 87 by 5; 123 by 4; 238 by 3; 527 by 5; 308 by 6; 516 by 7.

36. Divide 207 by 3; 304 by 8; 306 by 9; 315 by 9; 410 by 10; 264 by 11; 1728 by 12.

NOTE.—Young pupils may aid themselves, if necessary, by using the slate and pencil.

SECTION II.

INTRODUCTION TO FRACTIONS.

LESSON I.

Half, Third, and Fourth.

IF I divide an apple into two equal parts, what is one of these parts called? What are two of these parts called?

2. How many halves of an apple in one apple?

3. What is 1 half of 6?

Solution.—1 half of 6 is 3, because 2 times 3 are 6.

4. What is 1 half of 4? of 8? of 10? of 12?

5. What is 1 half of 14? of 16? of 18? of 20?

6. What is 1 half of 22? of 26? of 28? of 32?

7. What do we understand by one-half of anything?

8. If 1 pound of sugar costs 10 cents, what will 1 half of a pound cost?

9. Paul, having 20 apples, gave 1 half of them to his brother; how many did he give away?

10. Thompson bought 24 cows, and sold 1 half of them; how many did he sell?

11. Phebe had 40 peaches, and gave 1 half of them away: how many remained?

12. If I divide an apple into 3 equal parts, what is one of these parts called?

13. What are 2 and 3 of these parts called?

14. How many thirds in one apple?

15. What is one-third of anything?

16. What is 1 third of 6? of 9? of 12? of 15?

17. What is 1 third of 21? of 24? of 30? of 36?

18. James had 30 cents, and lost 1 third of them; how many did he lose?

19. Henry had 39 pears, and Thomas had 1 third as many; how many had Thomas?

20. Lucy had 21 pins, and gave Mary 1 third of them; how many did Mary receive?

21. A bought 42 cows, and sold 1 third of them to B; how many had he remaining?

22. What are 2 thirds of 9?

Solution.—1 third of 9 is 3; and if 1 third of 9 is 3, 2 thirds of 9 are 2 times 3, which are 6. Therefore, etc.

23. What are 2 thirds of 6? of 12? of 15? of 18?

24. What are 2 thirds of 24? of 30? of 27? of 33?

25. John had 21 cents, and gave 2 thirds of them to Sarah; how many cents did Sarah receive?

26. Having 27 peaches, I sold 2 thirds of them; how many did I sell?

27. Henry gave his sister 2 thirds of 33 oranges; how many did he retain?

28. Hiram lost 2 thirds of 36 dollars; how much money had he remaining?

29. If I divide an apple into 4 equal parts, what are 1, 2, 3, of these parts called?

30. How many fourths of an apple in a whole apple?

31. What is one-fourth of anything?

32. What is 1 fourth of 4? of 8? of 20? of 32?

33. What is 1 fourth of 12? of 24? of 16? of 48?

34. What are 2 fourths of 24? of 16? of 28? of 36?

35. What are 2 fourths of 12? of 20? of 40? of 48?

36. What are 3 fourths of 20? of 24? of 12? of 16?

37. Jacob, having 44 pens, sold 2 fourths of them to Joseph; how many pens did Joseph receive?

38. If a yard of cloth costs 8 dollars, what will 3 fourths of a yard cost?

39. Samson gave his brother 1 fourth and his sister 2 fourths of 28 oranges; how many did each receive?

40. A, having 24 plums, gave 1 half of them to B, and 1 third to C; how many had he left?

41. Harleigh is 24 years of age, and Townsend is 3 fourths as old; how old is Townsend?

42. A farmer had 36 sheep; A bought 1 half of them, and a dog killed 1 third; how many remained?

43. A merchant, having 40 barrels of flour, sold 3 fourths of them, and then bought 1 third as many as he sold; how many had he then?

LESSON II.

Fifth, Sixth, and Seventh.

IF you divide an orange into 5 equal parts, what are 1, 2, 3, and 4 of these parts called?

2. How many fifths in one orange? Define one-fifth.

3. What is 1 fifth of 10? of 25? of 15? of 30?

4. What are 2 fifths of 15? of 30? of 45? of 20?

5. What are 3 fifths of 10? of 30? of 25? of 55?

6. What are 4 fifths of 55? of 35? of 40? of 50?

7. Mary has 15 oranges, and Rachael has 2 fifths as many; how many has Rachael?

8. Susan's age is 25 years, and her sister is 4 fifths as old; how old is her sister?

9. Rowland is 35 years of age, and his wife is 3 fifths as old; how old is his wife?

10. A horse cost 100 dollars, and a sleigh cost 3 fifths as much; required the cost of the sleigh.

11. A man, having 40 sheep, lost 20, and found only 3 fifths of them; how many remained?

12. A man, having 50 cows, sold 4 fifths of them, and then bought 32 cows; how many had he then?

13. If you divide a melon into 6 equal parts, what are 1, 2, 3, 4, and 5 of these parts called?

14. How many sixths are there in a single thing?

15. What are 2 sixths of 24? of 18? of 36? of 60?

16. What are 3 sixths of 12? of 42? of 30? of 66?

17. What are 4 sixths of 6? of 36? of 48? of 54?

18. What are 5 sixths of 18? of 54? of 24? of 72?

19. Raub, having 48 pens, sold 3 sixths to Frescoln, and 2 sixths to Morgan; how many did he sell to both?

20. What will 5 sixths of 36 yards of cloth cost, at the rate of 2 dollars a yard?

21. Warren had 12 marbles, and Oliver had 5 sixths as many, lacking 4; how many had Oliver?

22. Dana, having 60 peaches, gave 2 sixths of them to Barton, and 3 sixths to Benton; how many remained?

23. If 1 yard of linen costs 5 sixths of 36 cents, how many yards can you buy for 60 cents?

24. 2 thirds of 30 dollars is 10 dollars less than A's money; required A's money.

25. Frazier, having 40 pens, gave Brown 10, and Seal 2 sixths of the remainder; how many had he left?

26. If a melon be divided into 7 equal parts, what are 1, 2, 3, 4, 5, and 6 of these parts called?

27. How many sevenths are there in one?

28. What is 1 seventh of 21? of 28? of 42? of 56?

29. What are 2 sevenths of 28? of 49? of 63? of 70?

30. What are 3 sevenths of 14? of 35? of 49? of 28?

31. What are 4 sevenths of 70? of 77? of 63? of 84?

32. What are 5 sevenths of 77? of 91? of 42? of 28?

33. What are 6 sevenths of 35? of 42? of 49? of 140?

34. A watch was bought for 70 dollars, and sold for 6 sevenths of its cost; required the loss.

35. If 1 half of 10 yards of cloth costs 10 dollars, what will 1 fifth of 10 yards cost?

36. A gave 70 dollars for a watch, and 3 sevenths as much for a chain, and sold them both for 90 dollars; required the loss.

37. 3 sevenths of 56 dollars is 6 dollars more than a

wagon-load of hay cost; what will 3 loads cost at the same rate?

38. Richard had 360 dollars, 1 third of which he spent for a horse, 1 fourth for a watch, and 1 sixth for a sleigh; how much had he remaining?

39. Mr. A, having 140 dollars, gave 3 sevenths of it to the poor, and lost 3 fourths of the remainder; how much then remained?

LESSON III.

Eighth and Ninth.

IF anything be divided into 8 equal parts, what is one of these parts called?

2. What are 2, 3, 4, 5, 6, and 7 of these parts called, and how many eighths in a unit?

3. What is 1 eighth of 24? 48? 72? 88?

4. What are 2 eighths of 32? 40? 56? 72?

5. What are 3 eighths of 16? 64? 80? 96?

6. What are 5 eighths of 8? 24? 48? 64?

7. 2 eighths of 24 are how many times 3?

8. 3 eighths of 40 are how many times 5?

9. 4 eighths of 80 are how many times 8?

10. 5 eighths of 56 are how many times 7?

11. 6 eighths of 64 are how many times 12?

12. 7 eighths of 72 are how many times 3?

13. 3 eighths of 32 are how many times 1 third of 12?

14. 6 eighths of 40 are how many times 1 fourth of 24?

15. 4 eighths of 48 are how many times 2 thirds of 18?

16. 7 eighths of 96 are how many times 3 fifths of 10?

17. 5 eighths of 56 are how many times 5 sixths of 42?

18. 2 thirds of 27 are how many times 3 fourths of 12?

19. If a single thing be divided into 9 equal parts, what are 1, 2, 3, 4, etc., of these parts called?

20. What are 2 ninths of 18? 27? 45? 36?

21. What are 3 ninths of 63? 72? 81? 27?

22. What are 4 ninths of 9? 36? 54? 81?

23. What are 5 ninths of 54? 72? 63? 27?

24. What are 6 ninths of 81? 18? 36? 90?

25. What are 7 ninths of 18? 99? 27? 108?

26. 3 times 6, and 2 thirds of 6, are how many?

27. 4 times 12, and 3 fourths of 12, are how many?

28. 5 times 10, and 3 fifths of 10, are how many?

29. 6 times 12, and 3 sixths of 12, are how many?

30. 5 times 7, and 4 sevenths of 7, are how many?

31. 9 times 8, and 5 eighths of 8, are how many?

32. 2 times 18, and 7 ninths of 18, are how many?

33. 2 ninths of 18 are how many times 2 thirds of 3?

34. 5 ninths of 27 are how many times 5 sixths of 6?

35. 6 ninths of 54 are how many times 4 fifths of 15?

36. 3 ninths of 72 are how many times 2 eighths of 16?

37. 7 eighths of 24 are how many times 7 eighths of 8?

38. A bought 15 horses, and sold 6 of them, and then lacked 4 of having 20; how many had he at first?

39. Hiram and Oliver had each 26 cents; after Hiram had given Oliver 10, and Oliver had given Hiram 6, how many had each?

40. A farmer, having 48 bushels of oats, sold 2 fourths of them to one man, and 1 fourth to another; how many bushels did he sell to each?

41. A bought 60 cows, and sold 1 third of them to B, and 3 fifths of the remainder to C; how many cows had he remaining?

LESSON IV.

Addition, Subtraction, etc.

HARRY gave 1 third of an apple to his brother, and 2 thirds to his sister; how much did he give away?

2. Matthew gave 2 fifths of a peach to Elias, and 3 fifths to Morris; how much did he give to both?

3. James gave 3 sevenths of a melon to Harry, and 4 sevenths to Harvey; how much did he give away?

4. Fanny ate 3 eighths of a quart of chestnuts yesterday, and 4 eighths to-day; how much did she eat in all?

5. Ella gave 1 fourth of a melon to Phœbe, 2 fourths to Carrie, and 3 fourths to Kate; how much did she give away?

6. Philip gave 2 sixths of a dollar to Jane, 3 sixths to Sarah, and 5 sixths to Eliza; how much did he give away?

7. Willie lost 7 fifths of a dollar, and had 9 fifths of a dollar remaining; how much had he at first?

8. Matthew lost 6 eighths of a dollar from one pocket, and 7 eighths from the other, and had 5 eighths remaining; how much had he at first?

9. Dora gave 3 ninths of a pound of raisins to Ella, and 7 ninths to Daisy, and then had 3 ninths remaining; how many had she at first?

10. Jane had 7 eighths of a pound of candies, and gave Maria 5 eighths of a pound; how many eighths remained?

11. Frank had 6 sevenths of a melon, and gave Abram 4 sevenths of a melon; how much remained?

12. Louisa, having 10 eighths of a dollar, gave Lizzie 7 eighths of a dollar; how much remained?

13. What is the difference between 5 sevenths, and the sum of 4 sevenths and 6 sevenths?

14. Sallie, having 24 pears, gave Beula 2 eighths and Amanda 3 eighths of them; how many remained?

15. Rufus, having 1 third of a quart of chestnuts, bought 4 thirds of a quart, and then sold 1 quart; what part of a quart remained?

16. Peter, having 5 sixths of a bushel of apples, sold 3 sixths, and then bought 2 sixths of a bushel; how many sixths had he then?

17. What is the difference between the sum of 3

eighths and 7 eighths, and the sum of 4 eighths and 5 eighths?

18. A bought 20 sheep, and sold 2 tenths of them to B, 3 tenths to C, and 4 tenths to D; how many sheep remained?

19. A lady, having 36 yards of tape, sold 5 ninths of it to one person, and 3 ninths to another; how much had she then?

20. Mariana had 3 fourths of a pint of nuts, Elva had twice as many, and Ezra 3 times as many; how many had they all?

21. A bought 4 ninths of a bushel of wheat, and B bought 3 times as much; how much did B buy?

22. At 7 fifths of a dollar each, how much will 5 turkeys cost?

23. Mary, having 11 fifths of a melon, gave 2 fifths to Sarah, and twice as much to Sophia; how much remained?

24. Cornell gave 3 times 3 sixths of an apple to Gray, and had 4 times 3 sixths remaining; how much had he at first?

25. What will 1 fifth of a yard of tape cost, at the rate of 20 fourths cents a yard? What will 3 fifths cost at the same rate?

26. Stanton, having 2 thirds of a dollar, found 1 half of 4 thirds of a dollar; how many thirds of a dollar had he then?

27. Mr. A bought 7 tenths of a barrel of sugar, and then sold 2 thirds of 6 tenths of a barrel; how much remained?

28. Thornton, having 8 sixths of a bushel of corn, bought 3 fourths of 20 sixths of a bushel; how much had he then?

29. Ferris lost 6 ninths of a dollar, and then, having found 3 ninths of a dollar, had 3 fourths of 8 ninths of a dollar remaining; how much had he at first?

LESSON V.

Arithmetical Analysis.

HOW much will 4 apples cost, if 3 apples cost 9 cents?

Solution.—If 3 apples cost 9 cents, 1 apple will cost 1 third of 9 cents, which is 3 cents; and if 1 apple costs 3 cents, 4 apples will cost 4 times 3 cents, which are 12 cents. Therefore, etc.

2. What will 5 lemons cost, at the rate of 3 for 12 cents?

3. If 3 pairs of shoes cost 6 dollars, how much will 5 pairs cost?

4. If 4 peaches are worth 8 cents, what are 8 peaches worth?

5. What are 10 oranges worth, if 8 oranges cost 16 cents?

6. If 7 pounds of meat cost 42 cents, what will 9 pounds cost?

7. What cost 11 barrels of flour, at the rate of 5 barrels for 30 dollars?

8. How far will a man travel in 12 days, at the rate of 36 miles in 4 days?

9. How many tons of hay will a drover feed in 11 weeks, at the rate of 10 tons in 5 weeks?

10. Required the value of 5 ducks, at the rate of 120 cents for 3 ducks.

11. Mary gave 10 cents for apples, at the rate of 3 cents for 9; how many did she buy?

12. Fanny paid 8 dollars for lace, at the rate of 5 dollars for 15 yards; how many did she buy?

13. Wilkinson walked 7 hours, at the rate of 12 miles in 4 hours; how far did he travel?

14. Robert gave 12 oranges for apples, at the rate of 3 oranges for 9 apples; how many apples did he get?

15. At the rate of 3 melons for 12 oranges, how many oranges can be bought for 10 melons?

16. If 6 men can mow 12 acres of grass in a day, how much can 8 men mow in the same time?

17. If 10 men can dig 30 rods of ditch in one day, how much can 12 men dig in the same time?

18. How long will it take 4 ladies to use a box of tea, if 6 ladies can use it in 12 days?

19. If 5 boys can do a piece of work in 16 days, how long will it take 20 boys to do it?

20. In what time will 8 girls pick a bushel of berries, if 4 girls can do it in 8 hours?

21. How many men will be required to build a boat in 6 days, if 3 men can do it in 12 days?

22. How many men can do as much work in 4 days as 8 men can do in 40 days?

23. If it requires 10 men 8 days to build a wall, how many men will be required to build it in 5 days?

24. If 5 men build a boat in 20 days, how many men will be required to do it in 1 fourth of the time?

25. If 7 yards of cashmere cost 21 dollars, what will 2 thirds of 15 yards cost?

26. Mary gave 7 apples for 21 chestnuts; at this rate how many chestnuts could she get for 8 apples?

27. If 8 lemons are worth 16 oranges, how many oranges can you buy for 10 lemons?

28. At the rate of 6 citrons for 18 melons, how many melons may be purchased for 11 citrons?

29. If 9 apples are worth 27 chestnuts, how many chestnuts may be had for 12 apples?

30. I gave 8 yards of muslin for 6 quarts of molasses; what did the molasses cost a quart, if 4 yards of muslin cost 48 cents?

31. Albert gave 9 bushels of corn for 2 barrels of flour; what was the corn worth a bushel, if 8 barrels of flour cost 72 dollars?

32. B bought 7 yards of cloth for 21 dollars, and gave 4 yards for apples worth 2 dollars per barrel; how many barrels of apples did he receive?

LESSON VI.

Arithmetical Analysis.

HOW much will one yard of tape cost, if 2 thirds of a yard cost 4 cents?

Solution.—If 2 thirds of a yard of tape cost 4 cents, 1 third of a yard will cost 1 half of 4 cents, which is 2 cents, and 3 thirds, or one yard, will cost 3 times 2 cents, which are 6 cents. Therefore, etc.

2. What will one box of soap cost, if 3 fourths of a box cost 6 dollars?

3. If 4 fifths of a box of tea cost 8 dollars, what will one box cost?

4. If 3 fifths of a yard of cloth cost 6 dollars, what will one yard cost?

5. What will two pounds of starch cost, if 5 sixths of a pound cost 10 cents?

6. What cost 2 barrels of flour, at the rate of 4 dollars for 4 sixths of a barrel?

7. What cost 3 yards of cloth, if 3 sevenths of a yard cost 6 dollars?

8. How far can A walk in 4 days, if in 5 sixths of a day he can walk 20 miles?

9. What cost 5 boxes of raisins, if 3 fifths of a box cost 6 dollars?

10. What is 5 times the distance to Lancaster, if 3 fourths of the distance is 3 miles?

11. How much will 4 bushels of apples cost, if 5 tenths of a bushel cost 50 cents?

12. Mary bought 9 pecks of beans, at the rate of 12 cents for 6 sevenths of a peck; required the cost.

13. How much will 5 tons of hay cost, if 10 dollars will buy 5 sixths of a ton?

14. Pelton bought 4 dozen eggs, at the rate of 8 cents for 2 thirds of a dozen; how much did they cost?

15. What is the value of 3 bushels of peaches, at the rate of 2 dollars for 2 thirds of a bushel?

16. If 3 fourths of a barrel of flour cost 6 dollars, what will 5 eighths of a barrel cost?

17. A watch cost 30 dollars, and 4 fifths of its cost is twice the cost of the chain; what was the cost of the chain?

18. B's horse cost 200 dollars, and 3 fifths of its cost is 4 times the cost of the sleigh; required the cost of the sleigh.

19. Elmina is 25 years old, and 4 fifths of her age is 4 years less than twice Elmira's age; required Elmira's age.

20. A merchant, having 20 barrels of flour, sold 3 fourths of it to A, and 3 fifths of the remainder to B; how much remained?

21. Think of a number, multiply it by 8, divide by 4, multiply by 3, divide by 6, add 20, subtract the number thought of, divide by 4, and name the result.

22. Think of a number, multiply by 12, divide by 3, multiply by 2, divide by 8, add 12, subtract the number thought of, divide by 4, and name the result.

23. Multiply 10 by 12, divide by 6, multiply by 5, divide by 2, multiply by 4, divide by 10, and name the result.

24. How many are 24 multiplied by 6, divided by 3, multiplied by 8, divided by 4, multiplied by 2, divided by 4, divided by 8?

LESSON VII.

Reducing Fractions.

HOW many thirds are there in 4?

Solution.—In 1 there are 3 thirds, and in 4 there are 4 times 3 thirds, which are 12 thirds. Therefore, etc.

2. How many thirds in 2? 3? 5? 7? 8?

3. How many fourths in 3? 5? 6? 4? 7?

4. How many fifths in 5? 4? 3? 2? 8?

5. How many sixths in 3? 2? 5? 6? 4?

6. How many sevenths in 2? 5? 4? 7? 9?

7. How many eighths in 3? 6? 4? 5? 7?

8. How many ninths in 8? 4? 6? 3? 10?

9. How many thirds in 3 and 2 thirds? in 4 and 1 third?

10. How many fifths in 4 and 3 fifths? in 6 and 3 fifths?

11. How many fourths in 2 and 1 fourth? in 7 and 3 fourths?

12. How many sevenths in 5 and 6 sevenths? in 3 and 4 sevenths?

13. How many sixths in 7 and 5 sixths? in 3 and 2 sixths?

14. How many eighths in 5 and 3 eighths? in 5 and 7 eighths?

15. If 5 yards of cloth cost 2 and 1 half dollars, what will six yards cost?

16. If 4 pears are worth 2 and 2 thirds cents, what are 7 pears worth?

17. What cost 10 peaches, at the rate of 4 and 1 half cents for 3 peaches?

18. If 11 ducks cost 4 and 2 fifths dollars, what will 12 ducks cost?

19. If 1 half of eight yards of cloth cost 3 and 1 fifth dollars, what will 9 yards cost?

20. If 2 thirds of 9 apples cost 4 and 4 fifths cents, what what will 3 fourths of 12 apples cost?

21. If 4 fifths of ten pounds of sugar cost 5 and 1 third dimes, what will ten pounds cost?

22. How many whole ones in 6 thirds?

Solution.—In *one* there are *three* thirds; hence, in *six* thirds there are as many ones as 3 is contained times in 6, which are 2. Therefore, in 6 thirds there are 2 ones.

23. How many ones in 6 halves? 9 thirds? 12 thirds?

24. How many ones in 12 fourths? 20 fourths? 8 fourths?

25. How many ones in 10 fifths? 12 sixths? 14 sevenths?

26. How many ones in 16 eighths? 21 sevenths? 24 eighths?

27. How many ones in 18 ninths? 15 thirds? 25 fifths?

28. How many ones in 28 sevenths? 36 ninths? 24 fourths?

29. How many ones in 15 thirds? 20 tenths? 33 elevenths?

30. If 2 apples cost 6 fifths of a cent, what will 5 apples cost?

31. What cost 5 pairs of shoes, if 4 pairs of shoes cost 12 fifths dollars?

32. How much are 8 pies worth, if 3 pies are worth 15 fourths dimes?

33. What cost 12 pineapples, if 3 pineapples cost 3 fourths of a dollar?

34. How much are 9 lamps worth, if 5 lamps are worth 10 thirds dollars?

35. Required the cost of 8 hats, if 6 hats cost 12 fourths of a dollar.

36. How much are 4 mirrors worth, if 7 mirrors are worth 14 halves dollars?

37. What cost 3 halves of a yard of linen, if 5 yards cost 10 ninths of a dollar?

38. What cost 1 half of 8 yards of cloth, if 1 half of 6 yards cost 1 third of 27 dollars?

39. What cost 1 half of 12 bushels of apples, if 1 third of 12 bushels cost 1 fourth of 12 dollars?

LESSON VIII.

Arithmetical Analysis.

1. 3 is 1 half of what number?

Solution.—If 1 half of some number is 3, 2 halves, or that number, is 2 times 3, which are 6. Therefore, etc.

2. 4 is 1 third of what number?

3. 6 is 1 fourth of what number?

4. 5 is 1 sixth of what number?

5. 6 is 1 half of what number?

6. 8 is 1 seventh of what number?

7. 9 is 1 fifth of what number?

8. 7 is 1 ninth of what number?

9. 5 is 1 seventh of what number?

10. 10 is 1 sixth of what number?

11. 9 is 1 third of what number?

12. 11 is 1 fourth of what number?

13. A is 10 years old, which is 1 fifth of B's age; required B's age.

14. An apple cost 3 cents, which is 1 fourth of the cost of a melon; required the cost of the melon.

15. A sheep cost 6 dollars, which is 1 fifth of the cost of a cow; what was the cost of the cow?

16. John has 20 marbles, which is 1 third of Henry's number; how many has Henry?

17. Mary's shawl cost 7 dollars, which is 1 fourth of the cost of her dress; required the cost of her dress.

18. Henry found 5 marbles, which is 1 third of what he had; how many had he then?

19. 6 is 1 half of 3 times what number?

20. 5 is 1 fourth of 2 times what number?

21. 8 is 1 third of 4 times what number?

22. 9 is 1 half of 6 times what number?

23. 7 is 1 sixth of 3 times what number?

24. 12 is 1 third of 9 times what number?

25. 11 is 1 sixth of 3 times what number?

26. Flora's cloak cost 10 dollars, which is 1 third of 6 times the cost of her dress; required the cost of her dress.

27. A watch was bought for 20 dollars, which is 1 third of 4 times what the chain cost; required the cost of the chain.

28. The head of a fish is 6 inches long, which is 1 fourth of 3 times the length of the body; what is the length of the body?

29. A boy lost 15 cents, which is 1 fourth of 5 times the money he had remaining; how much money had he at first?

30. 8 is 1 third of 1 half of what number?

31. 4 is 1 third of 1 fifth of what number?

32. 3 is 1 fifth of 1 fourth of what number?

33. 2 is 1 eighth of 1 fourth of what number?

34. 3 is 1 seventh of 1 sixth of what number?

35. 4 is 1 tenth of 1 fifth of what number?

36. 5 is 1 half of 1 seventh of what number?

37. Martin is 4 years old, and his age is 1 third of 1 fourth of his father's age; how old is his father?

38. Philip's vest cost 4 dollars, which is 1 half of 1 third of the cost of his coat; what was the cost of his coat?

39. Ella's bonnet cost 3 dollars, which is 1 fifth of 1 half the cost of her shawl; required the cost of the shawl.

40. A paid 10 dollars for a saddle, which is 1 fifth of 1 third of the cost of his horse; required the cost of the horse.

41. The head of a fish is 3 inches long, and the tail 5 inches, which is 1 half of 1 third of the length of the body; required the length of the fish.

LESSON IX.

Arithmetical Analysis.

1. 6 is 2 thirds of what number?

Solution.—If 2 thirds of some number is 6, 1 third of that number is 1 half of 6, which is 3, and 3 thirds, or that number, is 3 times 3, or 9. Therefore, etc.

2. 9 is 3 fourths of what number?

3. 6 is 2 thirds of what number?

4. 10 is 2 fifths of what number?

5. 12 is 4 sixths of what number?

6. 10 is 5 sevenths of what number?

7. 8 is 4 ninths of what number?

8. 9 is 3 fifths of what number?

9. 15 is 5 sixths of what number?

10. 10 is 5 eighths of what number?

11. 16 is 8 ninths of what number?

12. 14 is 7 thirds of what number?

13. Frank is 12 years old, and his age is 3 fifths of Fanny's age; how old is Fanny?

14. Augustus gave his brother 10 peaches, which is 2 thirds of all he had; how many had he?

15. A lady found 12 dollars, which is 4 sixths of what money she then had; how much had she at first?

16. Lester sold a cow for 24 dollars, which is 6 fifths of the cost of the cow; required its cost.

17. A farmer sold a colt for 30 dollars, and thereby gained 1 fifth of the cost of the colt; required the cost.

18. Frank lost 12 marbles, which is 2 fifths of what he had at first; how many remained?

19. Martin is 20 years of age, and 4 fifths of his age is twice his brother's age; what is the age of his brother?

20. 10 is 1 half of 4 fifths of what number?

21. 12 is 1 third of 6 sevenths of what number?

22. 16 is 2 fifths of 10 fourths of what number?

23. 15 is 5 sixths of 6 sevenths of what number?

24. 14 is 7 fourths of 4 thirds of what number?

25. 18 is 9 eighths of 4 sevenths of what number?

26. 20 is 5 fourths of 8 thirds of what number?

27. Thomas sold a book for 40 cents, which is 4 fifths of 5 sixths of the cost; required the cost.

28. Smiley sold his horse for 140 dollars, which is 7 eighths of 4 thirds of its value; required its value.

29. A's hat cost 6 dollars, which is 3 fourths of 4 fifths of the cost of his vest; required the cost of his vest.

30. 20 feet of a pole is in the water, which is 2 fifths of 5 sevenths of the length in the air; what is the length of the pole?

31. A pole is 30 feet in the air, which is 3 fifths of 2 fourths of the length of the pole; required the length of the pole.

32. A cow cost 24 dollars, which is 3 fourths of 4 fifths of the cost of the cow and a calf; what was the cost of the calf?

33. A's horse cost 200 dollars, and 4 fifths of the cost of the horse is 8 times the cost of the harness; required the cost of the harness.

34. A man has 24 geese, and 3 fourths of the number of geese equals 2 times the number of turkeys; how many turkeys has he?

35. A man sold his watch for 60 dollars, which is 5 fourths of 4 times what the chain cost, and the watch cost 3 times as much as the chain; what was the cost of each?

LESSON X.

Relation of Numbers.

1. 6 are how many times 1? 2? 3?

2. 8 are how many times 1? 2? 4?

3. 12 are how many times 2? 3? 6?

4. 20 are how many times 4? 5? 10?

5. 32 are how many times 4? 8? 16?
6. 30 are how many times 5? 6? 10?
7. 80 are how many times 4? 8? 20?
8. What is the relation of 8 to 2?

REMARK.—8 is 4 times 2.

9. What is the relation of 12 to 4? of 15 to 5?
10. What is the relation of 16 to 8? of 18 to 6?
11. What is the relation of 21 to 7? of 24 to 8?
12. What is the relation of 32 to 4? of 27 to 9?
13. What is the relation of 40 to 10? of 45 to 5?
14. What is the relation of 28 to 7? of 36 to 4?
15. If 4 yards of cloth cost 10 dollars, what will 8 yards cost?

Solution.—If 4 yards of cloth cost 10 dollars, 8 yards, which are 2 times 4 yards, will cost 2 times 10 dollars, or 20 dollars. Therefore, etc.

16. If three bunches of grapes cost 8 cents, what will 6 bunches cost?
17. If 6 combs cost 9 cents, what will 12 combs cost, at the same rate?
18. If 7 peaches cost 8 cents, what will 21 peaches cost, at the same rate?
19. If 5 pairs of shoes cost 9 dollars, what will 20 pairs cost, at the same rate?
20. How much will 42 primers cost, at the rate of 6 primers for 20 cents?
21. What will 56 inkstands cost, if 7 inkstands cost 5 dollars?
22. If 5 pitchers cost 3 dollars, what will 45 pitchers cost, at the same rate?
23. How far will a man travel in 48 days, if he travel 30 miles in 4 days?
24. If 6 men build 10 rods of wall in a given time, how many rods can 54 men build in the same time?

25. Hiram bought 6 pigs for 11 dollars; how many could he have bought for 44 dollars?

26. 2 men earn 12 dollars in 3 days; how much could they earn in 27 days?

27. If 5 peaches are worth 1 pear, how many pears are 30 peaches worth?

28. If 8 dollars will buy 5 gold pens, how many will 56 dollars buy?

29. If 6 stands cost 2 thirds of 12 dollars, what will 30 stands cost?

30. A gave 60 dollars for sheep, at the rate of 10 dollars for 3 sheep; how many did he purchase?

31. If 4 men can perform a piece of work in 18 days, how long will it require 12 men to do it?

32. If 6 men can build a boat in 10 fourths days, how long will it take 3 men to build it?

33. If 15 dollars is 3 eighths of what A earns in 5 days, how much will he earn in 15 days?

34. 18 men are 3 fifths of the number required to mow a field in 8 days; how many men would be required to mow it in 24 days?

35. 20 dollars is 4 dollars more than 2 thirds of 4 times what B paid for a chain, and his watch cost 5 times as much as the chain; required the cost of each.

LESSON XI.

Relation of Numbers.

1. 4 is what part of 8?

Solution.—4 is 1 half of 8, since 2 times 4 are 8.

2. 3 and 6 are what parts of 12?

3. 4 and 8 are what parts of 16?

4. 3 and 6 are what parts of 24?

5. 7 and 3 are what parts of 21?

6. 4 and 9 are what parts of 36?

7. What is the relation of 2 to 6? of 4 to 8?

8. What is the relation of 3 to 9? of 5 to 10?

9. What is the relation of 3 to 12? of 4 to 16?

10. What is the relation of 5 to 20? of 6 to 36?

11. What is the relation of 7 to 42? of 8 to 40?

12. What is the relation of 6 to 54? of 9 to 27?

13. If 6 apples cost 10 cents, what will 3 apples cost?

REMARK.—3 apples, one half of 6 apples, will cost one half of 10 cents, or 5 cents.

14. How much will 5 books cost, if 20 books cost 16 dollars?

15. How much will 3 knives cost, if 18 knives cost 24 dollars?

16. If 14 pencils cost 35 cents, what will 2 pencils cost, at the same rate?

17. If 10 peaches are worth 12 oranges, how many oranges are 5 peaches worth?

18. How much will 4 apples cost, if 16 apples cost 24 cents?

19. How much will 9 pigs cost, if 27 pigs cost 36 dollars?

20. What cost 7 tons of hay, if 56 tons cost 96 eagles?

21. If 42 sheep are sold for 108 dollars, what are 7 sheep sold for?

22. What cost 5 inkstands, if 15 inkstands cost 2 thirds of 18 dimes?

23. If 20 hens cost 10 dollars, what will 2 thirds of 6 hens cost?

24. If 3 fourths of 48 oranges cost 40 cents, what will 3 fourths of 12 oranges cost?

25. If A walked 132 miles in 33 days, how far did he walk in 3 days?

26. James bought 8 lamps, at the rate of 45 dollars a dozen; how much did they cost?

27. Mary, having 27 roses, gave 1 third of them to Sallie and 1 third to Annie; how many remained?

28. A worked 5 weeks for 7 dollars a week, and received in payment 12 bushels of wheat worth 1 dollar and 1 half a bushel; how much remains due A?

29. A farmer gave 10 bushels of rye, worth 6 dimes a bushel, for cloth worth 3 dollars a yard; how many yards did he receive?

30. Think of a number, multiply by 10, divide by 5, multiply by 3, divide by 6, add 30, subtract the original number, divide by 10, add 7, and the result is what?

LESSON XII.

Properties of Numbers.

WHAT numbers multiplied together will produce 4? 8? 10? 16? 12? 18? 24?

2. What numbers multiplied together will produce 15? 21? 28? 35? 36? 39? 48?

3. What numbers multiplied together will produce 40? 42? 45? 49? 50? 51? 52?

Numbers which can be produced by multiplying together other numbers, each of which is greater than a unit, are called *composite numbers.*

Numbers which cannot be produced by multiplying together two or more numbers, each of which is greater than a unit, are called *prime numbers.*

4. Tell which of the following numbers are prime, and which composite: 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23.

5. Name the prime and composite numbers in the following list: 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41.

The numbers which, when multiplied together, will produce a composite number, are called *factors* of that number.

6. What are the factors of 12? 20? 16? 33? 30? 24? 27? 18? 25? 32?

7. What are the factors of 9? 10? 14? 34? 36? 40? 48? 50? 56? 60? 63? 72?

When these factors are prime numbers, they are called *prime factors.*

8. What are the prime factors of 4? 6? 9? 12? 15? 18?

9. What are the prime factors of 10? 20? 21? 22? 24? 25? 27? 28? 30?

10. What are the prime factors of 32? 33? 35? 44? 45? 46? 48? 49? 50?

11. What are the prime factors of 52? 54? 55? 56? 57? 60? 64? 68? 70? 72? 75? 80?

12. Is a number exactly divisible by any number except its prime factors, or some product of them?

13. Name the divisors of 4; of 6; of 8; of 10; of 12; of 14; of 16; of 20.

14. What divisors are common to 4 and 6? To 8 and 10? To 6 and 9?

15. What divisors are common to 10 and 30? To 9 and 18? To 8 and 24?

16. What divisors are common to 9 and 27? To 10 and 20? To 16 and 24?

A divisor common to two or more numbers is called their *common divisor.*

17. What is a common divisor of 8 and 24? Of 9 and 15? Of 15 and 20?

18. What is a common divisor of 18 and 80? Of 16 and 32? Of 32 and 40?

The greatest divisor common to two or more numbers is called their *greatest common divisor.*

19. What is the greatest common divisor of 4 and 8? Of 8 and 24?

20. What is the greatest common divisor of 9 and 27? Of 16 and 24? Of 24 and 32?

A number which is one or more times another number is called a *multiple* of that number.

21. What is a multiple of 4? Of 3? Of 5? Of 6? Of 11?

22. What is a multiple of 7? Of 8? Of 9? Of 10? Of 12? Of 20?

A multiple common to two or more numbers is called their *common multiple.*

23. What is a common multiple of 2 and 3? Of 3 and 4? Of 4 and 5? Of 5 and 6?

24. What is a common multiple of 6 and 7? Of 4 and 6? Of 5 and 10? Of 9 and 12?

The least multiple common to two or more numbers is called their *least common multiple.*

25. What is the least common multiple of 4 and 6? Of 6 and 8? Of 8 and 10? Of 10 and 12?

26. What is the least common multiple of 8 and 12? Of 9 and 6? Of 9 and 12? Of 12 and 20?

If a number be multiplied by itself, the result is called the *square* of the number; if the square be multiplied by the number, the result is the *cube;* if the cube be multiplied by the number, the result is the *fourth power,* etc.

27. What is the square, cube, and fourth power of 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

The *square root* of a number is one of the two equal factors of that number; the cube root, one of the three equal factors; the fourth root, one of the four equal factors.

28. What is the square root of 1? 4? 9? 16? 25? 49? 81? 36? 64?

29. What is the cube root of 1? 8? 27? 64? 125? 343? 729? 512?

30. What is the fourth root of 1? 16? 81? 256? 625?

Define a prime number; composite number; factors; prime factors; common divisor; greatest common divisor; common multiple; least common multiple; square, cube, and fourth power; square, cube, and fourth roots.

LESSON XIII.

Arithmetical Analysis.

Arithmetical Analysis is a process of comparing numbers by their elements. In comparing, the *unit* is made the basis of the comparison. We reason to the unit and from the unit; the unit being the centre around which the logical process revolves.

The several cases of arithmetical analysis already considered will now be formally stated, and the process of reasoning illustrated. This lesson is to be explained to pupils rather than recited by them.

CASE I.—*To pass from a unit to any number.*

1. If *one* apple costs 3 cents, what will *four* apples cost?

Solution.—If *one* apple costs 3 cents, *four* apples, which are *four* times *one* apple, will cost 4 times 3 cents, or 12 cents.

REMARK.—Here the relation of *four* to *one*, of the *collection* to the *unit*, is given in the formation of the number as a collection of units: this relation is thus readily seen, and the transition easily made.

CASE II.—*To pass from any number to the unit.*

1. If four apples cost 12 cents, what will *one* apple cost?

Solution.—If *four* apples cost 12 cents, *one* apple, which is *one-fourth* of *four* apples, will cost *one-fourth* of 12 cents, or 3 cents.

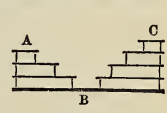
REMARK.—This is the reverse of the preceding case. The relation of *one* to *four*, of the *unit* to the *collection*, is immediately apprehended, and the transition readily made.

CASE III.—*To pass from a number to a number.*

1. If three apples cost 6 cents, what will four apples cost?

Solution.—If *three* apples cost 6 cents, *one* apple will cost $\frac{1}{3}$ of 6 cents, or 2 cents; and *four* apples will cost 4 times 2 cents, or 8 cents.

REMARK.—In this case we are to pass from the collection *three* to the collection *four*. In comparing *three* and *four*, their relation is not readily seen; but knowing the relation of *three* to the *unit*, and of the *unit* to *four*, we make the transition from 3 to 4 by passing through the unit. The unit thus becomes a basis of reference in the comparison, a kind of "stepping-stone" in the process.



This may be illustrated as follows: Suppose one standing at A, and wishing to pass over to C. Unable to step directly from A to C, he first steps down to the starting-point B, and then ascends to C. So in comparing numbers, when we cannot pass directly from the one to the other, we go down to the unit or starting-point of numbers, and then go up to the other number. These relations are intuitively apprehended, being presented in the formation of numbers. In the given problem we stand

three steps above the unit, and wish to go *four* steps above the unit. To do this we first *descend* the *three* steps, and then *ascend* the *four* steps.

CASE IV.—*To pass from a unit to a fraction.*

1. If one ton of hay costs 8 dollars, what will $\frac{3}{4}$ of a ton cost?

Solution.—If *one* ton of hay costs 8 dollars, *one fourth* of a ton will cost $\frac{1}{4}$ of $8, or $2; and *three fourths* of a ton will cost 3 times $2, or $6.

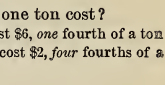
REMARK.—In this problem we are to pass from the unit to one of the equal divisions of the unit, and then to a collection of such equal divisions. In other words, we descend from the integral unit to the fractional unit, and then ascend among the fractional units. To illustrate: Suppose we are standing on the plane A, and wish to pass to the position C, we first take *four* steps down to B, and then *three* steps up to C.

CASE V.—*To pass from a fraction to a unit.*

1. If $\frac{3}{4}$ of a ton of hay cost $6, what will one ton cost?

Solution.—If *three* fourths of a ton of hay cost $6, *one* fourth of a ton will cost $\frac{1}{3}$ of $6, or $2; and if *one* fourth of a ton cost $2, *four* fourths of a ton, or one ton, will cost 4 times $2, or $8.

REMARK.—In this problem we are to pass from a collection of fractional units to the fractional unit, and then to the integral unit. To illustrate: Suppose we are standing at A, and wish to pass to C, we first step down *three* steps to B, and then step up *four* steps to C.

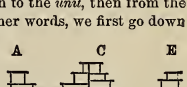


CASE VI.—*To pass from a fraction to a fraction.*

1. If $\frac{3}{4}$ of a ton of hay cost $15, what will $\frac{4}{5}$ of a ton cost?

Solution.—If *three* fourths of a ton cost $15, *one* fourth of a ton will cost $\frac{1}{3}$ of $15, or $5; and if *one* fourth of a ton cost $5, *four* fourths of a ton, or *one* ton, will cost 4 times 5, or $20. If *one* ton cost $20, *one fifth* of a ton will cost $\frac{1}{5}$ of $20, or $4, and *four* fifths of a ton will cost $4 \times \$4$, or $16.

REMARK.—In this problem we wish to compare the two fractions $\frac{3}{4}$ and $\frac{4}{5}$, but since we cannot perceive the relation between them directly, we must compare them through their relation to the unit. To do this we first go from *three* fourths to *one* fourth, then from *one* fourth to the *unit*, then from the unit to *one fifth*, and then to *four fifths*. In other words, we first go down from the *collection* of fractional units to the fractional *unit*, and then up to the *integral unit;* we then descend to the *other fractional unit*, and then ascend to the *number of fractional units* required. To illustrate: Suppose we are standing at A, and wish to pass to E, we cannot step directly from one point to the other, we therefore first pass from A three steps down to B, then from B four steps up to C, then from C five steps down to D, and then from D four steps up to E.



SECTION III.

TREATMENT OF FRACTIONS.

LESSON I.

Definitions.

ONE or more of the equal parts into which a unit is divided is called a *Fraction.*

The number of equal parts into which anything is divided, instead of being expressed by a word, may be represented by a figure beneath a line, thus:

$\frac{}{2}$ represents halves. $\frac{}{4}$ represents fourths.
$\frac{}{3}$ represents thirds. $\frac{}{5}$ represents fifths.

The number of fractional parts taken may be represented by a figure above the line, thus:

$\frac{2}{4}$ represents 2 fourths. $\frac{5}{7}$ represents 5 sevenths.
$\frac{3}{5}$ represents 3 fifths $\frac{7}{8}$ represents 7 eighths.
$\frac{4}{6}$ represents 4 sixths. $\frac{4}{9}$ represents 4 ninths.

The number written below the line is called the *Denominator;* it shows the number of equal parts into which the unit is divided.

The number written above the line is called the *Numerator;* it shows the number of equal parts taken.

A *Proper Fraction* is one whose value is less than a unit; as $\frac{3}{4}$, $\frac{5}{6}$.

An *Improper Fraction* is one whose value is equal to or greater than a unit; as $\frac{4}{4}$, $\frac{7}{7}$.

A *Mixed Number* consists of a whole number and a fraction; as $2\frac{1}{4}$, $3\frac{2}{5}$.

The *Reciprocal* of a number is a unit divided by that number; thus the reciprocal of 4 is $\frac{1}{4}$.

METHOD OF TREATMENT.—The method of treating fractions is by *analysis* and *induction*. We first solve each problem by *analysis*, and then lead the pupil to derive *methods of operations* or *rules* from these analyses by *inference* or *induction*.

CASES.—There are six general cases in the treatment of fractions, each of which includes several sub-cases. A full synopsis of the subject is presented in the following outline:

I. Reduction	1. Number to a Fraction. 2. Fraction to a Number. 3. To Higher Terms. 4. To Lower Terms. 5. Compound to Simple.
II. Addition	1. Denominators alike. 2. Denominators unlike.
III. Subtraction . . .	1. Denominators alike. 2. Denominators unlike.
IV. Multiplication . .	1. Fraction by a Number. 2. Number by a Fraction. 3. Fraction by a Fraction.
V. Division	1. Fraction by a Number. 2. Number by a Fraction. 3. Fraction by a Fraction,
VI. Relation	1. A Number to a Number. 2. A Fraction to a Number. 3. A Number to a Fraction. 4. A Fraction to a Fraction.

LESSON II.

Numbers to Fractions.

HOW many thirds in $4\frac{2}{3}$?

Solution.—In 1 there are $\frac{3}{3}$, and in 4 there are 4 times $\frac{3}{3}$, which are $\frac{12}{3}$; and $\frac{12}{3}$ plus $\frac{2}{3}$ are $\frac{14}{3}$. Therefore, in $4\frac{2}{3}$ there are $\frac{14}{3}$.

2. How many halves in $3\frac{1}{2}$? $2\frac{1}{2}$? $4\frac{1}{2}$? $6\frac{1}{2}$?

3. How many thirds in $3\frac{1}{3}$? $2\frac{2}{3}$? $4\frac{1}{3}$? $5\frac{2}{3}$?

4. How many fourths in $3\frac{1}{4}$? $4\frac{3}{4}$? $2\frac{2}{4}$? $7\frac{3}{4}$?
5. How many fifths in $1\frac{1}{5}$? $2\frac{2}{5}$? $3\frac{3}{5}$? $4\frac{4}{5}$?
6. How many sixths in $2\frac{1}{6}$? $3\frac{2}{6}$? $4\frac{3}{6}$? $5\frac{4}{6}$?
7. How many eighths in $2\frac{3}{8}$? $6\frac{1}{8}$? $7\frac{4}{8}$? $8\frac{5}{8}$?
8. How many sevenths in $3\frac{2}{7}$? $5\frac{1}{7}$? $4\frac{3}{7}$? $2\frac{5}{7}$?
9. How many ninths in $2\frac{2}{9}$? $3\frac{4}{9}$? $7\frac{5}{9}$? $6\frac{7}{9}$?
10. How many tenths in $5\frac{3}{10}$? $7\frac{5}{10}$? $3\frac{8}{10}$? $6\frac{7}{10}$?

11. In reducing $4\frac{2}{3}$ to thirds, by what do we multiply the 4? What do we do with the 2?

12. In reducing any mixed number to a fraction, by what do we multiply the whole number? What do we add to the product?

13. How, then, may we reduce a mixed number to a fraction without employing the analysis?

14. Reduce to improper fractions $2\frac{3}{4}$; $5\frac{2}{3}$; $6\frac{2}{5}$; $4\frac{1}{5}$.
15. Reduce to improper fractions $5\frac{2}{3}$; $2\frac{5}{6}$; $3\frac{2}{7}$; $8\frac{2}{3}$.
16. Reduce to improper fractions $4\frac{3}{4}$; $6\frac{2}{7}$; $5\frac{1}{8}$; $9\frac{1}{3}$.
17. Reduce to improper fractions $7\frac{2}{3}$; $8\frac{3}{4}$; $6\frac{5}{8}$; $9\frac{4}{5}$.

18. If 1 yard of tape costs $2\frac{1}{3}$ cents, what will 5 yards cost?

Solution.—$2\frac{1}{3}$ equals $\frac{7}{3}$. If one yard of tape costs $\frac{7}{3}$ cents, 5 yards will cost 5 times $\frac{7}{3}$ cents, which are $\frac{35}{3}$ cents.

19. How many fourths of a dollar will 7 baskets of peaches cost, at $2\frac{1}{4}$ dollars a basket?

20. If 4 boys can earn $6\frac{2}{3}$ dollars in a week, how much does each boy earn?

21. Carrie bought 5 pairs of shoes for $18\frac{3}{4}$ dollars; how much was that a pair?

22. How far will a man walk in 1 hour, at the rate of $19\frac{1}{2}$ miles in 7 hours?

23. If 3 girls earn $2\frac{1}{4}$ dollars in a day, how much could 5 girls earn, at the same rate?

24. If a man can earn $5\frac{1}{4}$ dollars in 3 days, how much can he earn in 5 days?

25. How long will it take a boy to plow 8 furrows, if it takes him $13\frac{3}{4}$ minutes to plow 5 furrows?

26. How far can Harry walk in 7 hours, if he can walk $6\frac{2}{5}$ miles in 2 hours?

27. How much cost 9 barrels of flour, if 2 barrels of flour cost $10\frac{2}{3}$ dollars?

28. If $2\frac{2}{3}$ yards of cloth cost 16 dollars, how much will one yard cost?

29. If $3\frac{1}{2}$ tons of coal cost 21 dollars, how much will one ton cost, at the same rate?

30. If 5 hats can be bought for $11\frac{1}{4}$ dollars, how much will 8 hats cost?

31. If $4\frac{1}{2}$ pounds of sugar cost 36 cents, how much must I pay for 7 pounds of sugar?

32. How much will 14 pounds of cloves cost, if $3\frac{1}{3}$ pounds can be bought for 50 cents?

33. If $2\frac{1}{7}$ yards of muslin cost 30 cents, how much will 10 yards cost, at the same rate?

LESSON III.

Fractions to Numbers.

HOW many ones are there in $\frac{14}{3}$?

Solution.—There are $\frac{3}{3}$ in 1, and in $\frac{14}{3}$ there are as many ones as 3 is contained times in 14, which are $4\frac{2}{3}$.

2. How many ones in $\frac{7}{2}$? $\frac{5}{2}$? $\frac{12}{3}$? $\frac{8}{3}$?

3. How many ones in $\frac{6}{3}$? $\frac{9}{3}$? $\frac{10}{3}$? $\frac{12}{8}$?

4. How many ones in $\frac{8}{4}$? $\frac{14}{4}$? $\frac{10}{4}$? $\frac{18}{4}$?

5. How many ones in $\frac{7}{5}$? $\frac{11}{3}$? $\frac{18}{6}$? $\frac{21}{7}$?

6. How many ones in $\frac{5}{4}$? $\frac{18}{9}$? $\frac{14}{6}$? $\frac{28}{14}$?

7. How many ones in $\frac{15}{5}$? $\frac{16}{8}$? $\frac{17}{8}$? $\frac{21}{9}$?

8. How many ones in $\frac{16}{9}$? $\frac{15}{7}$? $\frac{11}{4}$? $\frac{10}{6}$?

9. How many ones in $\frac{18}{7}$? $\frac{25}{6}$? $\frac{24}{10}$? $\frac{27}{9}$?

10. How many ones in $\frac{11}{4}$? $\frac{22}{4}$? $\frac{25}{10}$? $\frac{16}{11}$?

11. In reducing $\frac{14}{3}$ to a mixed number, by what do we divide the numerator 14?

12. In reducing any improper fraction to a whole or mixed number, by what do we divide the numerator?

13. How, then, can we reduce an improper fraction to a number without employing the analysis?

14. Reduce to mixed numbers $\frac{16}{5}$; $\frac{15}{2}$; $\frac{19}{9}$ and $\frac{25}{3}$.

15. Reduce to mixed numbers $\frac{14}{8}$; $\frac{15}{4}$; $\frac{13}{7}$ and $\frac{23}{6}$.

16. Reduce to mixed numbers $\frac{28}{5}$; $\frac{41}{6}$; $\frac{23}{8}$ and $\frac{44}{9}$.

17. Reduce to mixed numbers $\frac{24}{7}$; $\frac{25}{8}$; $\frac{26}{9}$ and $\frac{27}{10}$.

18. If 1 pound of sugar costs $6\frac{2}{3}$ cents, what will 8 pounds cost?

Solution.—If 1 pound of sugar costs $6\frac{2}{3}$ cents, 8 pounds will cost 8 times $6\frac{2}{3}$ cents; 8 times 6 cents are 48 cents; 8 times $\frac{2}{3}$ of a cent are $\frac{16}{3}$ of a cent, or $5\frac{1}{3}$ cents; 48 cents plus $5\frac{1}{3}$ cents are $53\frac{1}{3}$ cents. Therefore, etc.

19. At $6\frac{4}{5}$ dimes a bushel, what will 12 bushels of wheat cost?

20. What cost 8 barrels of apples, at the rate of $3\frac{2}{5}$ dollars a barrel?

21. How much will a man earn in a week, at the rate of $2\frac{3}{4}$ dollars a day?

22. At the rate of $18\frac{3}{9}$ cents a dozen, what will 3 dozen eggs cost?

23. If 2 apples cost $4\frac{2}{3}$ cents, how much will 5 apples cost?

24. If three loads of hay cost $5\frac{1}{4}$ dollars, what will 6 loads of hay cost?

25. What cost 5 pounds of butter, at the rate of $3\frac{1}{8}$ dollars for 10 pounds?

26. What cost 7 barrels of apples, at the rate of 9 dollars for $2\frac{1}{4}$ barrels?

27. How many chestnuts can I buy for 7 cents, if 14 chestnuts cost $3\frac{1}{2}$ cents?

28. What cost $5\frac{3}{8}$ pounds of beef, if 2 pounds cost 32 cents?

29. How far will a man drive in $5\frac{1}{3}$ hours, at the rate of 21 miles in $3\frac{1}{2}$ hours?

30. A vessel sailed 23 miles in $4\frac{3}{5}$ hours; how far did she sail in 12 hours?

31. A kite arose 48 rods in $3\frac{3}{7}$ minutes; how far at this rate will it ascend in 6 minutes?

32. What cost 18 pounds of meat, at the rate of $4\frac{1}{2}$ pounds for $3\frac{3}{8}$ dimes?

33. If $\frac{2}{5}$ of a ton of hay is worth $4\frac{4}{5}$ dollars, what cost $7\frac{2}{3}$ tons of hay?

34. If a man walks 7 miles in $2\frac{1}{3}$ hours, how far will he walk in $4\frac{2}{3}$ hours?

LESSON IV.

Reduction to Higher Terms.

HOW many fourths in $\frac{1}{2}$?

Solution.—In 1 there are $\frac{4}{4}$, and in $\frac{1}{2}$ there are $\frac{1}{2}$ of $\frac{4}{4}$, or $\frac{2}{4}$. Therefore, etc.

2. How many sixths in $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{2}$?

3. How many eighths in $\frac{1}{2}$? $\frac{1}{4}$? $\frac{2}{4}$? $\frac{3}{4}$?

4. How many tenths in $\frac{1}{2}$? $\frac{1}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

5. How many twelfths in $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{4}{6}$?

6. How many fourteenths in $\frac{1}{2}$? $\frac{2}{7}$? $\frac{4}{7}$? $\frac{5}{7}$?

7. How many fifteenths in $\frac{2}{3}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{3}$?

8. How many sixteenths in $\frac{1}{4}$? $\frac{3}{4}$? $\frac{2}{8}$? $\frac{5}{8}$?

9. How many eighteenths in $\frac{2}{3}$? $\frac{4}{6}$? $\frac{3}{9}$? $\frac{8}{9}$?

10. How many twentieths in $\frac{4}{5}$? $\frac{3}{4}$? $\frac{7}{10}$? $\frac{3}{5}$?

11. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths.

12. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$ to twentieths.

13. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{9}{10}$, and $\frac{14}{15}$ to thirtieths.

14. Since $\frac{2}{3} = \frac{4}{6}$, by what number may you multiply both numerator and denominator of $\frac{2}{3}$ to obtain $\frac{4}{6}$?

15. Since $\frac{3}{4} = \frac{9}{12}$, by what number may you multiply both numerator and denominator of $\frac{3}{4}$ to obtain $\frac{9}{12}$?

16. By what must you multiply the numerator and denominator of $\frac{3}{5}$ to reduce it to tenths?

17. By what must you multiply both numerator and denominator of $\frac{3}{4}$ to reduce it to twentieths?

18. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{5}$ to twentieths.

19. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{6}$ to thirtieths.

20. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to twelfths.

21. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$ to eighteenths.

22. Reduce $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, and $\frac{5}{8}$ to sixteenths.

When fractions have the same denominator they are said to have a *common denominator*.

23. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

Solution.—A common denominator for 3ds and 4ths is 12ths. In 1 there are $\frac{12}{12}$, and in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$, and in $\frac{2}{3}$, etc.

24. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to a common denominator.

25. Reduce $\frac{1}{4}$ and $\frac{1}{5}$ to a common denominator.

26. Reduce $\frac{1}{3}$ and $\frac{1}{5}$ to a common denominator.

27. Reduce $\frac{1}{5}$ and $\frac{1}{6}$ to a common denominator.

28. Reduce $\frac{2}{3}$ and $\frac{3}{6}$ to a common denominator.

29. Reduce $\frac{1}{2}$ and $\frac{1}{7}$ to a common denominator.

30. Reduce $\frac{2}{3}$ and $\frac{3}{8}$ to a common denominator.

31. Reduce $\frac{2}{5}$ and $\frac{2}{6}$ to a common denominator.

32. Reduce $\frac{2}{4}$ and $\frac{3}{10}$ to a common denominator.

33. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to a common denominator.

34. Reduce $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ to a common denominator.

35. Reduce $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ to a common denominator.

36. Reduce $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{10}$ to a common denominator.

37. If $2\frac{1}{2}$ yards of silk cost 20 dimes, what will 4 yards cost?

38. If $2\frac{1}{6}$ yards of tape cost 13 cents, what will 3 yards cost?

39. Mary lost 20 roses, which is $\frac{2}{3}$ as many as she then had; how many had she at first?

40. John found 60 cents, which is $\frac{5}{4}$ of $\frac{1}{2}$ of what he then had; how much had he at first?

41. What cost $3\frac{1}{5}$ pounds of sugar, if $2\frac{1}{2}$ pounds of sugar cost 25 cents?

42. Henry gave his sister 20 cents, which is $\frac{4}{5}$ of what Henry had at first, and $\frac{1}{2}$ of what his sister now has; how much had each at first?

LESSON V.

Reduction to Lower Terms.

HOW many thirds are there in $\frac{4}{6}$?

Solution.—In 1 there are $\frac{6}{6}$, and in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{6}{6}$, which are $\frac{2}{6}$. If there are $\frac{2}{6}$ in $\frac{1}{3}$, in $\frac{4}{6}$ there are as many thirds as 2 is contained times in 4, which are 2; hence in $\frac{4}{6}$ there are $\frac{2}{3}$.

2. How many halves in $\frac{2}{4}$? $\frac{6}{4}$? $\frac{8}{4}$? $\frac{10}{4}$?

3. How many thirds in $\frac{2}{6}$? $\frac{4}{6}$? $\frac{6}{9}$? $\frac{8}{12}$?

4. How many fourths in $\frac{1}{2}$? $\frac{6}{8}$? $\frac{9}{12}$? $\frac{10}{8}$?

5. How many sixths in $\frac{10}{12}$? $\frac{9}{18}$? $\frac{2}{8}$? $\frac{8}{12}$?

6. How many eighths in $\frac{4}{16}$? $\frac{12}{16}$? $\frac{8}{16}$? $\frac{12}{24}$?

7. How many fifths in $\frac{8}{10}$? $\frac{6}{15}$? $\frac{12}{20}$? $\frac{16}{20}$?

8. How many sevenths in $\frac{10}{14}$? $\frac{8}{14}$? $\frac{6}{21}$? $\frac{12}{21}$?

9. How many ninths in $\frac{12}{18}$? $\frac{16}{18}$? $\frac{15}{27}$? $\frac{18}{27}$?

10. How many tenths in $\frac{16}{20}$? $\frac{21}{30}$? $\frac{24}{40}$? $\frac{25}{50}$?

11. Since $\frac{4}{6}=\frac{2}{3}$, by what may we divide both numerator and denominator of $\frac{4}{6}$ to produce $\frac{2}{3}$?

12. By what number must we divide both numerator and denominator of $\frac{4}{8}$ to reduce it to fourths?

13. By what must we divide both numerator and denominator of $\frac{5}{10}$ to reduce it to halves?

14. Reduce $\frac{6}{10}$ to fifths; and $\frac{3}{12}$ to fourths.

15. Reduce $\frac{7}{14}$ to halves; and $\frac{9}{12}$ to fourths.

16. Reduce $\frac{8}{16}$ to fourths; and $\frac{6}{9}$ to thirds.

17. Reduce $\frac{16}{20}$ to fifths; and $\frac{8}{24}$ to sixths.

18. Reduce $\frac{9}{21}$ to sevenths; and $\frac{16}{36}$ to ninths.

19. Reduce $\frac{15}{24}$ to eighths; and $\frac{24}{30}$ to tenths.

20. Reduce $\frac{28}{42}$ to tenths; and $\frac{35}{60}$ to twelfths.

21. Reduce $\frac{25}{45}$ to ninths; and $\frac{35}{65}$ to elevenths.

When a fraction cannot be reduced to an equivalent one having a less denominator, it is said to be in its *lowest terms.*

22. Reduce $\frac{10}{12}$ and $\frac{9}{15}$ to their lowest terms.

23. Reduce $\frac{12}{15}$ and $\frac{8}{12}$ to their lowest terms.

24. Reduce $\frac{16}{20}$ and $\frac{20}{36}$ to their lowest terms.

25. Reduce $\frac{15}{45}$ and $\frac{24}{36}$ to their lowest terms.

26. Reduce $\frac{21}{42}$ and $\frac{24}{48}$ to their lowest terms.

27. Reduce $\frac{25}{35}$ and $\frac{27}{36}$ to their lowest terms.

28. If 6 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of 3 times the same number?

29. If 8 is $\frac{4}{5}$ of some number, what is $\frac{1}{5}$ of 2 times the same number?

30. 4 times 50 years is 10 years less than 10 times the age of James; how old is he?

31. If 4 horses eat 2 tons of hay in 8 weeks, how long will it require 5 horses to eat the same?

32. If 2 men can build a boat in 16 days, how long will it require 8 men to build it?

33. If it required $8\frac{3}{4}$ yards of cloth to make 2 coats, how much will it require to make 9 coats?

34. 42 dollars is $\frac{6}{7}$ of all the money A has, and B has 3 times as much; how much money has B?

35. A gave B 48 cents, and $\frac{5}{6}$ of this is 4 times as much as he had remaining; how much had he at first?

36. Amanda, having 50 pins, lost $\frac{4}{5}$ of them, and then found $\frac{2}{3}$ as many as remained; how many had she then?

37. A watch cost $90, which is $\frac{3}{5}$ of 10 times what the chain cost; required the cost of both.

38. How many lemons will pay for 7 melons, if 6 lemons are worth $4\frac{4}{5}$ melons?

39. Mary gave Lilly 24 pins, which is $\frac{3}{5}$ of what Lilly then had, and $\frac{3}{5}$ of what Mary had remaining; how many had each then?

LESSON VI.

Addition of Fractions.

WHAT is the sum of $\frac{1}{3}$ and $\frac{2}{5}$?

2. What is the sum of $\frac{2}{4}$ and $\frac{3}{4}$?
3. What is the sum of $\frac{4}{6}$ and $\frac{3}{6}$?
4. What is the sum of $\frac{2}{5}$ and $\frac{4}{5}$?
5. How many fourths in $\frac{1}{2}$ and $\frac{3}{4}$?
6. How many eighths in $\frac{3}{4}$ and $\frac{3}{8}$?
7. How many tenths in $\frac{1}{2}$ and $\frac{1}{5}$?
8. How many twelfths in $\frac{1}{4}$ and $\frac{1}{3}$?
9. How many fifteenths in $\frac{2}{3}$ and $\frac{3}{5}$?
10. How many sixteenths in $\frac{3}{4}$ and $\frac{5}{8}$?
11. How many eighteenths in $\frac{2}{3}$ and $\frac{5}{6}$?
12. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

Solution.—$\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$; $\frac{8}{12}$ plus $\frac{9}{12}$ are $\frac{17}{12}$, which equals $1\frac{5}{12}$. Therefore, etc.

What is the sum

13. Of $\frac{1}{2}$ and $\frac{1}{3}$?
14. Of $\frac{1}{3}$ and $\frac{1}{4}$?
15. Of $\frac{1}{4}$ and $\frac{1}{5}$?
16. Of $\frac{2}{3}$ and $\frac{3}{4}$?
17. Of $\frac{2}{3}$ and $\frac{2}{5}$?
18. Of $\frac{2}{5}$ and $\frac{3}{4}$?
19. Of $\frac{3}{4}$ and $\frac{3}{8}$?
20. Of $\frac{2}{4}$ and $\frac{2}{7}$?
21. Of $\frac{5}{6}$ and $\frac{4}{5}$?
22. Of $\frac{1}{4}$ and $\frac{1}{5}$?
23. Of $\frac{2}{5}$ and $\frac{3}{7}$?
24. Of $\frac{2}{3}$ and $\frac{3}{5}$?
25. Of $\frac{5}{6}$ and $\frac{4}{7}$?
26. Of $\frac{3}{7}$ and $\frac{3}{8}$?
27. Of $2\frac{1}{2}$ and $3\frac{1}{3}$?
28. Of $3\frac{1}{3}$ and $4\frac{1}{4}$?
29. Of $2\frac{2}{3}$ and $1\frac{3}{4}$?
30. Of $3\frac{1}{4}$ and $2\frac{3}{5}$?
31. Of $6\frac{2}{5}$ and $5\frac{3}{8}$?
32. Of $4\frac{1}{6}$ and $5\frac{1}{7}$?
33. Of $6\frac{1}{5}$ and $5\frac{1}{8}$?
34. Of $7\frac{2}{3}$ and $8\frac{3}{7}$?
35. Of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$?
36. Of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$?
37. Of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

38. James earned $\frac{1}{4}$ of a dollar on Monday, and $\frac{1}{5}$ of a dollar on Tuesday; how much did he earn in both days?

39. Jane bought a slate for $\frac{1}{4}$ of a dollar, and an arithmetic for $\frac{5}{8}$ of a dollar; what was the cost of both?

40. A man, having $\frac{2}{5}$ of a barrel of flour, bought $\frac{3}{4}$ of a barrel; how much had he then?

41. Mary, having $\frac{1}{2}$ of a dozen of pins, found $\frac{5}{6}$ of a dozen; how many pins had she then?

42. Jane, having a certain sum of money, found $\frac{3}{4}$ as much; what part of the sum had she then?

43. If $\frac{1}{2}$ of a number, increased by $\frac{1}{3}$ of the number, equals 50, what is the number?

44. Sarah, having 40 roses, gave $\frac{1}{2}$ of them to her sister and $\frac{1}{8}$ of them to her brother; how many remained?

45. James' money, increased by its $\frac{4}{5}$, equals 90 cents; how much money had James?

46. $\frac{2}{3}$ of a certain number, increased by $\frac{3}{4}$ of the same number, equals 34; required the number.

47. Fanny's number of roses, increased by $\frac{1}{2}$ and $\frac{1}{3}$ of her number, equals 55; how many roses has she?

48. $\frac{2}{5}$ of A's money, increased by $\frac{3}{4}$ of his money, equals 69 dollars; how much money has A?

49. Peter, having $\frac{3}{8}$ of a certain sum of money, found $\frac{1}{2}$ of the same sum, and then had $21; how much was the sum?

50. A gave 24 dollars for a watch, and $\frac{1}{2} + \frac{2}{3}$ of this is 4 times what he paid for the chain; required the cost of the chain?

LESSON VII.

Subtraction of Fractions.

WHAT is the difference between $\frac{6}{7}$ and $\frac{3}{7}$?

2. What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$?

3. What is the difference between $\frac{5}{6}$ and $\frac{5}{9}$?

4. What is the difference between $2\frac{3}{4}$ and $1\frac{1}{4}$?

5. What is the difference between $3\frac{1}{3}$ and $2\frac{2}{3}$?

6. How many fourths in $\frac{1}{2}$ minus $\frac{1}{4}$?

7. How many eighths in $\frac{7}{8}$ minus $\frac{3}{4}$?

8. How many sixths in $\frac{1}{2}$ minus $\frac{1}{3}$?

9. How many twelfths in $\frac{5}{6}$ minus $\frac{3}{4}$?

10. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

Solution.—$\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{2}{3}$ is equal to $\frac{8}{12}$; $\frac{9}{12}$ minus $\frac{8}{12}$ equals $\frac{1}{12}$. Therefore, etc.

Subtract

11. $\frac{1}{3}$ from $\frac{1}{2}$.
12. $\frac{1}{5}$ from $\frac{1}{4}$.
13. $\frac{2}{3}$ from $\frac{3}{4}$.
14. $\frac{1}{6}$ from $\frac{1}{4}$.
15. $\frac{2}{5}$ from $\frac{2}{4}$.
16. $\frac{2}{3}$ from $\frac{4}{5}$.
17. $\frac{2}{5}$ from $\frac{3}{6}$.
18. $\frac{3}{5}$ from $\frac{3}{4}$.
19. $\frac{3}{4}$ from $\frac{8}{9}$.
20. $\frac{1}{6}$ from $\frac{1}{5}$.
21. $\frac{1}{7}$ from $\frac{1}{6}$.
22. $\frac{1}{3}$ from $\frac{5}{6}$.
23. $\frac{2}{5}$ from $\frac{2}{3}$.
24. $\frac{1}{2}$ from $\frac{5}{7}$.
25. $\frac{1}{8}$ from $\frac{1}{7}$.
26. $\frac{1}{9}$ from $\frac{1}{2}$.
27. $\frac{2}{7}$ from $\frac{2}{3}$.
28. $\frac{3}{8}$ from $\frac{6}{8}$.
29. $2\frac{1}{2}$ from $3\frac{1}{4}$.
30. $3\frac{1}{5}$ from $4\frac{1}{4}$.
31. $2\frac{1}{4}$ from $3\frac{1}{5}$.
32. $3\frac{1}{5}$ from $5\frac{1}{3}$.

33. A man owned $\frac{3}{4}$ of a boat, and sold $\frac{1}{3}$ of the boat; what part of the boat did he still own?

34. A pole is in the mud, air, and water; if $\frac{3}{5}$ of it is in the mud and water, how much is in the air?

35. If $\frac{1}{2}$ of a pole is in the air, $\frac{1}{3}$ in the water, and the rest in the mud, how much is in the mud?

36. Maggie Wilson's age, diminished by its $\frac{1}{4}$, is 15 years; how old is she?

37. The difference between $\frac{2}{3}$ of my money and $\frac{3}{4}$ of my money is 9 dollars; how much money have I?

38. Mr. Smith bought 50 yards of trecot cloth, and sold $\frac{3}{5}$ of it; how many yards remained?

39. Maria owes a store bill of $\frac{3}{5}$ of a dollar; if she hands the clerk $\frac{3}{4}$ of a dollar, how much change should she receive?

40. In an orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{3}$ bear peaches, and the remainder bear pears; what part bear pears?

41. A boy, having 36 marbles, lost $\frac{3}{4}$ of them, and then found $\frac{5}{6}$ as many as he had at first; how many had he then?

42. \$40 is 4 times what A paid for a chain, and the cost of the chain is $\frac{1}{8}$ of the cost of his watch; required the cost of the watch.

What is the value

43. Of $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$?

44. Of $\frac{1}{2}+\frac{3}{4}-\frac{1}{3}$?

45. Of $\frac{1}{4}+\frac{2}{5}-\frac{1}{2}$?

46. Of $\frac{1}{2}+\frac{1}{6}-\frac{1}{4}$?

47. Of $\frac{1}{3}+\frac{1}{6}-\frac{1}{2}$?

48. Of $2\frac{1}{2}+4\frac{1}{5}-4\frac{1}{4}$?

49. Of $1\frac{1}{3}+1\frac{1}{4}-2\frac{1}{5}$?

50. Of $3\frac{3}{8}+2\frac{3}{4}-5\frac{1}{2}$?

51. \$60 is $\frac{3}{5}$ of what B gave for a horse, and the cost of the horse, increased by its 3 fifths, is 5 times what he paid for a sleigh; required the cost of the sleigh.

LESSON VIII.

Compound to Simple Fractions.

WHAT is $\frac{1}{3}$ of 4?

Solution.—$\frac{1}{3}$ of 1 is $\frac{1}{3}$, and if $\frac{1}{3}$ of 1 is $\frac{1}{3}$, $\frac{1}{3}$ of 4 is 4 times $\frac{1}{3}$, which are $\frac{4}{3}$. Therefore $\frac{1}{3}$ of 4 is $\frac{4}{3}$ of one.

2. What is $\frac{1}{3}$ of 5? $\frac{1}{4}$ of 6?

3. What is $\frac{1}{5}$ of 7? $\frac{1}{6}$ of 9?

4. What is $\frac{1}{2}$ of 5? $\frac{1}{4}$ of 10?

5. What is $\frac{1}{7}$ of 9? $\frac{1}{8}$ of 20?

6. What is $\frac{2}{4}$ of 6? $\frac{3}{4}$ of 10?

7. What is $\frac{2}{5}$ of 3? $\frac{5}{6}$ of 14?

8. What is $\frac{2}{7}$ of 4? $\frac{3}{8}$ of 15?

9. Mary has 6 dollars, and her brother has $\frac{3}{5}$ as much; how much has her brother?

10. A has 20 cents, and $\frac{1}{3}$ of his money equals 4 times B's money; how much money has B?

11. James is 7 years old, and $\frac{4}{9}$ of his age equals $\frac{7}{9}$ of Mary's age; how old is Mary?

12. A watch cost \$21, and $\frac{4}{5}$ of its cost is $\frac{7}{10}$ of the cost of the chain; required the cost of both.

13. A cask contains 5 gallons, and $\frac{6}{7}$ of its contents is $\frac{10}{14}$ of the contents of another cask; required the contents of the second cask.

14. What is $\frac{2}{3}$ of $\frac{6}{8}$?

Solution.—$\frac{1}{3}$ of $\frac{6}{8}$ is $\frac{2}{8}$, and if $\frac{1}{3}$ of $\frac{6}{8}$ is $\frac{2}{8}$, $\frac{2}{3}$ of $\frac{6}{8}$ are 2 times $\frac{2}{8}$, which are $\frac{4}{8}$, or $\frac{1}{2}$. Therefore, etc.

What is

15. $\frac{2}{5}$ of $\frac{6}{7}$?
16. $\frac{2}{7}$ of $\frac{7}{8}$?
17. $\frac{3}{6}$ of $\frac{6}{9}$?
18. $\frac{4}{5}$ of $\frac{18}{20}$?
19. $\frac{3}{5}$ of $\frac{15}{16}$?
20. $\frac{2}{7}$ of $\frac{14}{16}$?
21. $\frac{3}{5}$ of $2\frac{1}{2}$?
22. $\frac{5}{6}$ of $2\frac{2}{5}$?
23. $\frac{3}{4}$ of $3\frac{1}{5}$?
24. $\frac{2}{5}$ of $3\frac{3}{4}$?
25. $\frac{2}{3}$ of $1\frac{1}{3}$?
26. $\frac{2}{3}$ of $\frac{3}{4}$ of $1\frac{1}{7}$?

27. $\frac{3}{5}$ of \$40 is 2 times what A gave for a bureau; what was the cost of the bureau?

28. $\frac{5}{7}$ of $\frac{14}{25}$ of a dollar is $\frac{1}{10}$ of the cost of a watch; how much did the watch cost?

29. B has 27 marbles, and $\frac{2}{3}$ of B's number equals $\frac{2}{5}$ of C's number; how many marbles has C?

30. Henry's hat cost $\frac{4}{5}$ of an eagle, which is $\frac{2}{3}$ of $\frac{3}{5}$ of the cost of his coat; required the cost of the coat?

31. If there are 48 chestnuts in a pint, how many does each of two boys receive, if A receives $\frac{6}{8}$ of a pint, and B receives $\frac{2}{3}$ as many as A?

32. Mary bought $\frac{4}{6}$ of a paper of needles, which is $\frac{8}{9}$ of what Sarah bought; how many did each purchase, provided there are 24 needles in a paper?

33. A has 40 fruit trees, $\frac{4}{10}$ of which bear apples, $\frac{1}{2}$ of the remainder bear pears, and the rest bear peaches; how many trees of each kind has he?

34. Janson's age, diminished by its $\frac{1}{4}$ and $\frac{1}{5}$, is 22 years, and his age is $\frac{4}{5}$ of his uncle's age; required the age of each.

LESSON IX.

Compound to Simple Fractions.

WHAT is $\frac{1}{3}$ of $\frac{1}{4}$?

Solution.—$\frac{1}{3}$ of $\frac{1}{4}$ is *one* of the *three* equal parts into which $\frac{1}{4}$ may be divided; if each fourth is divided into three equal parts, 4 fourths or the unit will be divided into 4 times 3, or 12 equal parts; hence, each part is $\frac{1}{12}$ of a unit. Therefore, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

2. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{4}$?

3. What is $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{6}$?

4. What is $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{7}$?

5. What is $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{7}$? $\frac{1}{4}$ of $\frac{1}{7}$?

6. What is $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{6}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{9}$?

7. What is $\frac{1}{6}$ of $\frac{1}{12}$? $\frac{1}{8}$ of $\frac{1}{9}$? $\frac{1}{7}$ of $\frac{1}{10}$?

8. Mary, having $\frac{1}{5}$ of a pie, gave $\frac{1}{3}$ of it to Hannah; what part of a pie did Hannah receive?

9. Philo, having $\frac{1}{4}$ of an orange, gave $\frac{1}{5}$ of it to Peter; what part of an orange did Peter receive?

10. A had $\frac{1}{5}$ of a dollar, and gave $\frac{1}{5}$ of it to B; what part of a dollar did B receive?

11. Since $\frac{1}{3}$ of $\frac{1}{4}$ equals $\frac{1}{12}$, how may the same result be obtained without the analysis?

12. Since $\frac{1}{4}$ of $\frac{1}{5}$ equals $\frac{1}{20}$, how may we obtain the same result without the analysis?

NOTE.—Problem 1st may also be solved thus: $\frac{1}{4}$ equals $\frac{3}{12}$, and $\frac{1}{3}$ of $\frac{3}{12}$ is $\frac{1}{12}$, hence, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$. This is simpler than the solution given, but since it does not show *the reason why* $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, we prefer the other solution.

13. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{7}$?

14. What is $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{3}$? $\frac{1}{8}$ of $\frac{1}{9}$?

15. What is $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{7}$ of $\frac{1}{7}$? $\frac{1}{9}$ of $\frac{1}{5}$? $\frac{1}{10}$ of $\frac{1}{11}$?

16. A man, owning $\frac{1}{8}$ of a farm, sold $\frac{1}{6}$ of it to his neighbor; what part of the farm did he sell?

17. Susan bought $\frac{1}{5}$ of a cake, and gave Eliza $\frac{1}{3}$ of it; how much did Eliza receive?

18. Mary is 8 years old, and her age is $\frac{1}{2}$ of $\frac{1}{3}$ of her mother's age; how old is her mother?

19. A man, owning $\frac{1}{7}$ of the stock of a bank, sold $\frac{1}{4}$ of it; what part of his stock did he retain?

20. Carlo, finding $\frac{1}{5}$ of a pound of meat, let Towser eat $\frac{1}{4}$ of it; what part of a pound did Carlo eat?

21. A penholder cost 12 cents, and $\frac{2}{3}$ of its cost is $\frac{1}{3}$ of $\frac{1}{4}$ of the cost of the pen; required the cost of the pen.

22. What is $\frac{2}{3}$ of $\frac{4}{5}$?

Solution.—$\frac{1}{5}$ of $\frac{1}{5}$ is $\frac{1}{15}$; and if $\frac{1}{5}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$, which are $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{4}{15}$, or $\frac{8}{15}$. Therefore $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

23. What is $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{1}{5}$ of $\frac{4}{6}$?

24. What is $\frac{1}{4}$ of $\frac{5}{7}$? $\frac{1}{6}$ of $\frac{5}{8}$? $\frac{1}{7}$ of $\frac{5}{6}$?

25. What is $\frac{1}{5}$ of $\frac{6}{7}$? $\frac{1}{8}$ of $\frac{7}{9}$? $\frac{1}{6}$ of $\frac{4}{12}$?

26. What is $\frac{2}{3}$ of $\frac{3}{6}$? $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{4}{5}$ of $\frac{6}{8}$?

27. What is $\frac{3}{5}$ of $\frac{4}{6}$? $\frac{5}{6}$ of $\frac{5}{7}$? $\frac{3}{4}$ of $\frac{3}{6}$?

28. What is $\frac{2}{5}$ of $\frac{4}{7}$? $\frac{7}{10}$ of $\frac{5}{8}$? $\frac{5}{9}$ of $\frac{7}{10}$?

29. Since $\frac{2}{3}$ of $\frac{4}{5}$ equals $\frac{8}{15}$, in what manner may we obtain the same result by omitting the analysis?

30. How, then, shall we find a fractional part of a fraction without going through with the analysis?

31. What is $\frac{3}{5}$ of $\frac{3}{9}$? $\frac{2}{5}$ of $\frac{2}{7}$? $\frac{3}{4}$ of $\frac{3}{5}$?

32. What is $\frac{5}{7}$ of $\frac{2}{3}$? $\frac{6}{8}$ of $\frac{3}{4}$? $\frac{7}{6}$ of $\frac{3}{7}$?

33. What is $\frac{3}{4}$ of $\frac{8}{6}$? $\frac{4}{5}$ of $\frac{10}{8}$? $\frac{3}{7}$ of $\frac{14}{9}$?

34. A boy had $\frac{3}{4}$ of a dollar, and gave away $\frac{2}{5}$ of it; how much did he give away?

35. A man, owning $\frac{2}{5}$ of the stock of a bank, sold $\frac{3}{5}$ of it; how much stock did he sell?

36. Bowman, having $\frac{2}{7}$ of a melon, gave $\frac{2}{3}$ of it to Martin; what part of $\frac{2}{7}$ remained?

37. Having $\frac{3}{4}$ of a bushel of apples, I gave $\frac{3}{4}$ of them to Westlake; what part of a bushel remained?

38. Eva received $\frac{4}{5}$ of her father's money, and spent $\frac{5}{6}$ of it; how much remained?

39. Having lost $\frac{1}{4}$ of my money, I found $\frac{1}{2}$ of what I lost, and then had \$70; how much had I at first?

40. Harding lost $\frac{6}{9}$ of his money, and then found as much as $\frac{1}{6}$ of the remainder; what part of his money did he then have?

41. Annie, having $\frac{2}{5}$ of a pound of candies, shared them equally with 5 of her schoolmates; what part of a pound did each receive?

42. A squirrel fell $\frac{5}{6}$ of the distance from the top of a tree to the ground, and then ascended $\frac{1}{4}$ of the distance he was from the ground; what part of the whole distance was he from the ground?

LESSON X.

Multiplication of Fractions.

HOW many are 4 times $\frac{3}{8}$?

Solution.—4 times $\frac{3}{8}$ are $\frac{12}{8}$, which equals $\frac{3}{2}$, or $1\frac{1}{2}$. Therefore, etc.

2. How many are 3 times $\frac{3}{5}$? 3 times $\frac{2}{5}$?
3. How many are 4 times $\frac{6}{8}$? 3 times $\frac{2}{6}$?
4. How many are 7 times $\frac{3}{14}$? 5 times $\frac{4}{10}$?
5. How many are 4 times $\frac{8}{12}$? 6 times $\frac{7}{12}$?
6. How many are 3 times $\frac{6}{9}$? 4 times $\frac{2}{6}$?
7. How many are 8 times $\frac{2}{4}$? 8 times $\frac{3}{6}$?
8. How many are 5 times $\frac{3}{6}$? 5 times $\frac{7}{6}$?
9. How many are 3 times $\frac{7}{5}$?

Solution.—3 times $\frac{1}{3}$ equals $\frac{3}{3}$, or $\frac{1}{1}$; if 3 times $\frac{1}{3}$ equals $\frac{1}{1}$, 3 times $\frac{7}{3}$ equals 7 times $\frac{1}{1}$, or $\frac{7}{1}$; hence 3 times $\frac{7}{3}$ equals $\frac{7}{1}$.

How many are

10. 4 times $\frac{3}{8}$?
11. 3 times $\frac{8}{9}$?
12. 5 times $\frac{11}{15}$?
13. 5 times $\frac{9}{10}$?
14. 6 times $\frac{18}{24}$?
15. 7 times $\frac{18}{21}$?
16. 6 times $\frac{11}{18}$?
17. 7 times $\frac{12}{21}$?
18. 8 times $\frac{12}{24}$?
19. 9 times $\frac{24}{27}$?

20. Since 3 times $\frac{7}{9}$ equals $\frac{7}{3}$, how may this result be obtained by omitting the analysis?

21. How, then, may a fraction be multiplied by a number which will divide the denominator?

22. How do we multiply a fraction by a number that will not divide the denominator?

23. What is the effect of dividing the denominator of a fraction by a number?

How many are

24. 4 times $\frac{5}{8}$?

25. 6 times $\frac{5}{6}$?

26. 7 times $\frac{3}{14}$?

27. 5 times $\frac{19}{10}$?

28. 8 times $\frac{10}{16}$?

29. 9 times $\frac{15}{27}$?

30. 8 times $\frac{24}{32}$?

31. 6 times $\frac{18}{36}$?

32. 4 times $\frac{15}{16}$?

33. 3 times $2\frac{4}{6}$?

34. 5 times $2\frac{2}{5}$?

35. 7 times $3\frac{1}{7}$?

36. How much will $4\frac{2}{3}$ yards of muslin cost, at the rate of 6 cents a yard?

37. If you give to 8 boys each $2\frac{3}{4}$ apples, how many apples will it require?

38. How many dollars does that man give away who gives to each of 10 beggars $\frac{4}{5}$ of a dollar?

39. Mary gave to each of 12 poor children $\frac{5}{6}$ of a pie, and had 3 pies remaining; how many pies had she?

40. How much will 10 pounds of sugar cost, at the rate of $\frac{4}{5}$ of a dime a pound?

41. How much will 9 inkstands cost, at the rate of 2 for $\frac{4}{6}$ of a dollar?

42. If 4 apples cost $\frac{3}{4}$ of a dime, what will 16 apples cost, at the same rate?

43. What will 7 quarts of beans cost, if 4 quarts cost $\frac{4}{7}$ of a dollar?

44. If a man walk $3\frac{1}{5}$ miles in 2 hours, how far will he walk in 1 day of 10 hours?

45. 5 times $4\frac{4}{5}$ miles is 3 times the distance from Paoli to Oakland; required the distance.

46. 6 times $2\frac{2}{3}$ miles is $\frac{7}{10}$ of the distance from Oakland to Christiana; what is the distance?

47. 7 times $3\frac{1}{7}$ miles is $5\frac{1}{2}$ times the distance from Columbia to Marietta; what is the distance?

48. Peter gave $\frac{1}{3}$ of his marbles to Samuel, and 2 times $\frac{1}{4}$ of them to Anson; how many had he at first, if he gave away 14 marbles?

LESSON XI.

Division of Fractions.

HOW many times is 2 contained in $\frac{4}{5}$?

Solution.—1 is contained in $\frac{4}{5}$, $\frac{4}{5}$ times; and if 1 is contained in $\frac{4}{5}$, $\frac{4}{5}$ times, 2 is contained in $\frac{4}{5}$, $\frac{1}{2}$ of $\frac{4}{5}$ times, or $\frac{2}{5}$ times. Therefore, etc.

2. How many times is 3 contained in $\frac{3}{4}$? In $\frac{4}{5}$?

3. How many times is 4 contained in $\frac{8}{9}$? In $\frac{5}{6}$?

4. How many times is 5 contained in $\frac{5}{6}$? In $\frac{2}{3}$?

5. How many times is 6 contained in $\frac{6}{7}$? In $\frac{3}{5}$?

6. How many times is 7 contained in $\frac{7}{8}$? In $\frac{5}{8}$?

7. Since $\frac{4}{5} \div 2$ equals $\frac{2}{5}$, how may we divide a fraction by a number which will divide the numerator?

8. Since $\frac{5}{6} \div 2$ equals $\frac{5}{12}$, how may we divide a fraction by a number which will not divide the numerator?

9. If a yard of cloth costs \$4, how much can you buy for $\frac{3}{4}$ of a dollar?

10. Mary gave $\$2\frac{1}{2}$ for silk, at the rate of \$5 a yard; how much did she buy?

11. How long will it take to walk $\frac{6}{7}$ of a mile, at the rate of 3 miles an hour?

12. How long will it take to earn $\$4\frac{1}{2}$, at the rate of \$6 a week?

13. How many times is $\frac{2}{3}$ contained in 4?

Solution.—1 is contained in 4, 4 times; and if 1 is contained in 4, 4 times, $\frac{1}{3}$ is contained in 4, 3 times 4 times, which are 12 times, and 2 thirds is contained in 4, $\frac{1}{2}$ of 12 times, or 6 times. Therefore, etc.

14. How many times is $\frac{2}{3}$ contained in 2? In 3?
15. How many times is $\frac{2}{5}$ contained in 3? In 5?
16. How many times is $\frac{3}{7}$ contained in 2? In 4?
17. How many times is $\frac{2}{3}$ contained in 5? In 7?
18. How many times is $\frac{5}{6}$ contained in 4? In 5?
19. How many times is $\frac{6}{5}$ contained in 2? In 4?
20. By what method may we derive the results obtained above, without the analysis?
21. If a yard of cloth costs $\frac{3}{8}$ of a dollar, how many yards can you buy for 12 dollars?
22. If 1 quart of nuts costs $\frac{4}{5}$ of a dime, how many quarts can be bought for 1 dollar?
23. How many yards of tape can be bought for 11 cents, if 3 yards cost $2\frac{1}{5}$ cents?
24. How much will 7 peaches cost, at the rate of 3 peaches for $4\frac{1}{2}$ cents?
25. If 5 pints of milk cost 12 cents, how many pints can you purchase for 25 cents?
26. If $2\frac{1}{2}$ barrels of apples cost 15 dollars, how many barrels can be bought for 12 dollars?
27. How much will $2\frac{2}{5}$ oranges cost, at the rate of 4 oranges for $5\frac{1}{3}$ cents?
28. If $3\frac{1}{3}$ boxes of starch cost $\$5\frac{1}{4}$, what will 10 boxes cost, at the same rate?
29. If 3 men can do a piece of work in $6\frac{2}{3}$ days, how long will it take 12 men to do the same?
30. How many times is $\frac{2}{3}$ contained in $\frac{4}{5}$?

Solution.—1 is contained in $\frac{3}{4}$, $\frac{3}{4}$ times; and if 1 is contained in $\frac{3}{4}$, $\frac{3}{4}$ times, $\frac{1}{3}$ is contained in $\frac{3}{4}$, 3 times $\frac{3}{4}$ times, or $\frac{9}{4}$ times, and $\frac{2}{3}$ is contained in $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{9}{4}$ times, or $\frac{9}{8}$ times. Therefore, etc.

31. How many times is $\frac{2}{3}$ contained in $\frac{5}{8}$? In $\frac{3}{8}$?
32. How many times is $\frac{3}{4}$ contained in $\frac{3}{7}$? In $\frac{5}{6}$?
33. How many times is $\frac{5}{6}$ contained in $\frac{3}{4}$? In $\frac{4}{6}$?
34. How many times is $\frac{2}{3}$ contained in $\frac{5}{8}$? In $\frac{7}{7}$?
35. How many times is $\frac{4}{7}$ contained in $\frac{5}{8}$? In $\frac{6}{8}$?
36. How many times is $\frac{4}{5}$ contained in $\frac{8}{9}$? In $\frac{7}{10}$?

37. How many times is $\frac{3}{8}$ contained in $\frac{3}{7}$? In $\frac{5}{8}$?

38. How many times is $\frac{5}{6}$ contained in $\frac{10}{12}$? In $\frac{15}{18}$?

39. How many times is $\frac{2}{4}$ contained in $\frac{3}{5}$?

Another Solution.—$\frac{2}{4}$ is equal to $\frac{10}{20}$, and $\frac{3}{5}$ is equal to $\frac{12}{20}$; $\frac{10}{20}$ is contained as many times in $\frac{12}{20}$ as 10 is contained in 12, which is $\frac{12}{10}$, or $\frac{6}{5}$ times. Therefore, etc.

40. How many times is $\frac{2}{3}$ contained in $\frac{2}{4}$? In $\frac{3}{7}$?

41. How many times is $\frac{3}{4}$ contained in $\frac{2}{5}$? In $\frac{4}{7}$?

42. How many times is $\frac{3}{5}$ contained in $\frac{3}{4}$? In $\frac{2}{3}$?

43. How many times is $\frac{5}{6}$ contained in $\frac{2}{4}$? In $\frac{3}{7}$?

44. How many times $\frac{3}{4}$ is $\frac{3}{2}$? $\frac{5}{6}$? $\frac{3}{8}$? How many times $\frac{3}{5}$ is $\frac{2}{7}$? $\frac{3}{8}$? $\frac{9}{10}$?

45. Since in dividing by a fraction we see that we multiply by its denominator and divide by its numerator, what rule may we infer for dividing by a fraction?

46. If a yard of muslin costs $\frac{3}{5}$ of a dime, how many yards can you purchase for $\frac{7}{8}$ of a dime?

47. If a gallon of vinegar costs $\frac{2}{5}$ of a dollar, how many gallons can you buy for $2\frac{1}{2}$ dollars?

48. A boy divided 14 apples equally among his companions, giving to each $3\frac{1}{2}$ apples; required the number of his companions.

49. Mrs. Brown exchanged 20 pounds of butter, at 15 cents a pound, for calico, worth $12\frac{1}{2}$ cents a yard; how many yards did she receive?

50. A lady distributed 29 dimes equally among some poor children, giving to each $5\frac{4}{5}$ dimes; how many children were there?

51. B bought 6 yards of ribbon, worth $5\frac{1}{3}$ cents a yard; how many apples, worth $1\frac{1}{3}$ cents each, will be required to pay for it?

52. Harry bought 8 bushels of potatoes, worth $\$\frac{5}{8}$ a bushel, and paid for them with eggs, worth $\$\frac{2}{5}$ a dozen; how many eggs did it take?

LESSON XII.

Relation of Fractions.

WHAT part of 5 is 3?

Solution.—1 is $\frac{1}{5}$ of 5; and if 1 is $\frac{1}{5}$ of 5, 3 is 3 times $\frac{1}{5}$ of 5, or $\frac{3}{5}$ of 5. Therefore, etc.

2. What part of 4 is 3? Of 6 is 3?

3. What part of 5 is 2? Of 8 is 2?

4. What part of 6 is 4? Of 5 is 6?

5. What part of 7 is 8? Of 6 is 9?

6. What part of 8 is 4? Of 4 is 8?

7. If 8 apples cost 12 cents, what will 6 apples cost, at the same rate?

Solution.—If 8 apples cost 12 cents, 6 apples, which are $\frac{6}{8}$ or $\frac{3}{4}$ of 8 apples, will cost $\frac{3}{4}$ of 12 cents, which are 9 cents.

8. If 9 sheep cost 21 dollars, what will 12 sheep cost, at the same rate?

9. James has \$8, and Mary has \$6; what part of James's money equals Mary's money?

10. A watch cost \$40, and a chain cost \$12; what part of the cost of the watch equals the cost of the chain?

11. A has \$20, and B has $\frac{3}{5}$ as much, plus \$3; what part of A's money equals B's money?

12. What part of 2 is $\frac{3}{4}$?

Solution.—1 is $\frac{1}{2}$ of 2, and $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$ of 2, which is $\frac{1}{8}$ of 2, and $\frac{3}{4}$ is 3 times $\frac{1}{8}$, which are $\frac{3}{8}$ of 2. Therefore $\frac{3}{4}$ is $\frac{3}{8}$ of 2.

13. What part of 3 is $\frac{2}{5}$? Of 2 is $\frac{3}{4}$?

14. What part of 4 is $\frac{3}{5}$? Of 5 is $\frac{3}{5}$?

15. What part of 4 is $\frac{4}{6}$? Of 7 is $\frac{5}{8}$?

16. What part of 9 is $\frac{3}{7}$? Of 5 is $\frac{1}{2}$ of $\frac{3}{4}$?

17. What part of 6 is $\frac{2}{3}$? Of 7 is $\frac{2}{5}$ of $\frac{3}{2}$?

18. What part of 2 is $\frac{3}{5}$? Of 5 is $\frac{4}{5}$ of $\frac{5}{6}$?

19. A book cost \$4, and a slate \$$\frac{3}{4}$; what part of the cost of the book equals the cost of the slate?

20. A has \$5, and B has \$2½; what part of A's money equals B's money?

21. A hat was bought for \$2, and sold at a gain of \$¾; what part of the cost equals the gain?

22. A man can walk a certain distance in 6 hours; what part of the distance can he walk in 2½ hours?

23. Sarah bought 5 yards of silk, and used 3⅓ yards; what part of the silk then remained?

24. A bought 6 oranges, and B bought 4 oranges; what did B pay for his oranges, if A paid for his 26 cents?

25. What part of $\frac{2}{3}$ is $\frac{4}{5}$?

Solution.—$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{2}{3}$, or *one*, is 3 times $\frac{1}{2}$, or $\frac{3}{2}$ of $\frac{2}{3}$. Since *one* is $\frac{3}{2}$ of $\frac{2}{3}$, $\frac{1}{5}$ is $\frac{1}{5}$ of $\frac{3}{2}$, which is $\frac{3}{10}$ of $\frac{2}{3}$, and $\frac{4}{5}$ is 4 times $\frac{3}{10}$, which are $\frac{12}{10}$, or $\frac{6}{5}$ of $\frac{2}{3}$. Therefore, $\frac{4}{5}$ is $\frac{6}{5}$ of $\frac{2}{3}$.

What part

26. Of $\frac{4}{7}$ is $\frac{2}{5}$?

27. Of $\frac{3}{6}$ is $\frac{2}{3}$?

28. Of $\frac{4}{5}$ is $\frac{5}{6}$?

29. Of $\frac{6}{7}$ is $\frac{3}{8}$?

30. Of $\frac{2}{5}$ is $\frac{3}{4}$?

31. Of $\frac{3}{5}$ is $\frac{3}{4}$?

32. Of $\frac{3}{4}$ is $\frac{5}{6}$?

33. Of $\frac{5}{6}$ is $\frac{3}{4}$?

34. Of $\frac{5}{7}$ is $2\frac{1}{2}$?

35. Of $\frac{6}{7}$ is $\frac{2}{3}$ of $\frac{6}{7}$?

36. Of $\frac{7}{8}$ is $\frac{3}{4}$ of $\frac{5}{8}$?

37. Of $\frac{9}{10}$ is $\frac{4}{5}$ of $\frac{7}{8}$?

NOTE.—These problems may also be solved thus: $\frac{2}{3}$ equals $\frac{12}{18}$, $\frac{4}{5}$ equals $\frac{12}{15}$; and $\frac{12}{18}$ is the same part of $\frac{12}{15}$ that 12 is of 10, and 12 is $\frac{12}{10}$ or $\frac{6}{5}$ of 10.

38. A man owned $\frac{2}{3}$ of a vessel, and sold $\frac{1}{4}$ of the vessel; what part of his share did he sell?

39. A had $\frac{3}{4}$ of a dollar, and spent $\frac{2}{5}$ of a dollar; what part of his money did he spend?

40. A merchant bought $\frac{5}{6}$ of a cargo of oranges, and sold $\frac{3}{4}$ of them; what part of the cargo did he sell?

41. Floy has 40 cents, Evie has 60 cents, and 20 cents is the same part of Eddie's money that Floy's money is of Evie's; how many cents has Eddie?

42. Edwin and Emma each had \$2; Edwin spent $\frac{1}{4}$ of his, and Emma spent $\frac{1}{5}$ of hers; what part of Emma's money then equaled Edwin's?

LESSON XIII.

Miscellaneous.

A COW cost \$24, and $\frac{4}{5}$ of the cost of the cow is $\frac{2}{3}$ of the cost of a horse; required the cost of the horse.

2. A chain cost \$15, and $\frac{1}{2}$ of its cost is $\frac{3}{4}$ of the cost of a watch; required the cost of the watch.

3. How many apples does that man give away who gives to 5 girls each $\frac{5}{6}$ of an apple?

4. If a yard of cloth costs $\frac{3}{4}$ of a dollar, how many yards can be bought for 9 dollars?

5. How many yards of muslin can be bought for \$6, if 2 yards cost $\frac{3}{4}$ of a dollar?

6. If $\frac{1}{2}$ of an apple is worth $\frac{1}{3}$ of a cent, how much is $\frac{1}{5}$ of an apple worth?

7. Francis, after losing $\frac{3}{5}$ of his money, found that \$12 was $\frac{3}{4}$ of what remained; how much money had he?

8. If 3 men can do a piece of work in $3\frac{1}{3}$ days, how long will it take 5 men to do it?

9. If 7 men can do a piece of work in $2\frac{1}{3}$ days, how long will it require 6 men to do it?

10. If a yard of muslin costs $\frac{2}{3}$ of a dime, how much can you buy for $\frac{5}{6}$ of a dime?

11. A shared 8 apples with his companions, giving to each $\frac{4}{5}$ of an apple; required the number of companions.

12. How many bushels of corn, worth $5\frac{3}{4}$ shillings a bushel, can be bought for 34 shillings?

13. How many apples will pay for 10 peaches, if 5 apples are worth $8\frac{1}{3}$ peaches?

14. Mary shared 21 dimes with her schoolmates, giving to each $2\frac{1}{3}$ dimes; how many schoolmates had she?

15. 16 is $\frac{4}{5}$ of how many times 5?

16. 18 is $\frac{3}{4}$ of how many times 8?

17. 25 is $\frac{5}{8}$ of how many times 10?

18. 15 is $\frac{5}{7}$ of how many times 3?

19. 20 is $\frac{4}{5}$ of how many times $\frac{1}{2}$ of 10?

20. 24 is $\frac{3}{4}$ of how many times $\frac{2}{3}$ of 12?

21. 28 is $\frac{4}{7}$ of how many times $\frac{1}{3}$ of 21?
22. 30 is $\frac{5}{6}$ of how many times $\frac{3}{4}$ of 12?
23. 18 is $\frac{3}{8}$ of how many times $\frac{6}{7}$ of 14?
24. 40 is $\frac{4}{5}$ of how many times $\frac{5}{6}$ of 30?
25. 36 is $\frac{4}{8}$ of how many times $\frac{3}{5}$ of 15?
26. 60 is $\frac{8}{9}$ of how many times $\frac{5}{7}$ of 14?
27. $\frac{1}{2}$ of 16 is how many times $\frac{1}{3}$ of 12?
28. $\frac{2}{3}$ of 30 is how many times $\frac{2}{5}$ of 10?
29. $\frac{3}{4}$ of 40 is how many times $\frac{5}{7}$ of 21?
30. $\frac{4}{5}$ of 45 is how many times $\frac{3}{5}$ of 15?
31. $\frac{6}{9}$ of 42 is how many times $\frac{6}{7}$ of 14?
32. $\frac{7}{8}$ of 48 is how many times $\frac{3}{8}$ of 16?
33. $\frac{5}{8}$ of 80 is how many times $\frac{2}{5}$ of 25?
34. $\frac{5}{9}$ of 72 is how many times $\frac{5}{4}$ of 16?

35. A's horse cost \$200, and $\frac{4}{5}$ of this is twice the cost of his sleigh, and the sleigh cost 4 times as much as his harness; required the cost of each.

36. B's wedding-coat cost \$40, and $\frac{3}{4}$ of this is twice the cost of his vest, and also three times the cost of his hat; what was the cost of each and of all?

37. B bought 4 yards of silk, worth \$1$\frac{2}{3}$ a yard, and paid for it with cloth, worth \$1$\frac{1}{3}$ a yard; how many yards of cloth did it take?

38. The distance from Paoli to Christiana is 24 miles, and $\frac{2}{3}$ of this distance is $\frac{4}{5}$ of the distance from Christiana to Lancaster; what is the distance to Lancaster?

39. The distance from Columbia to Rockville is 30 miles, and $\frac{2}{3}$ of this distance is $\frac{2}{5}$ of the distance from Columbia to Newport; required the distance to Newport.

40. The distance from Conewago to Duncannon is 3 miles, and $\frac{2}{3}$ of this distance is $\frac{1}{5}$ of the distance from Conewago to Mexico; what is the distance to Mexico?

41. Hannah's wedding-dress cost \$50, and $\frac{4}{5}$ of this is 4 times the cost of her bonnet, and also $\frac{2}{3}$ of the cost of her cloak; required the cost of the bonnet and cloak, respectively.

LESSON XIV.

Principles of Fractions.

WE have derived the principles of the following propositions by *induction* from analytical processes; we shall now proceed to establish their truth by rigid demonstration.

PROP. 1.—*Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.*

PROP. 2.—*Dividing the numerator of a fraction by any number divides the value of the fraction by that number.*

PROP. 3.—*Multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

PROP. 4.—*Dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.*

PROP. 5.—*Multiplying both numerator and denominator of a fraction by any number does not change the value of the fraction.*

PROP. 6.—*Dividing both numerator and denominator of a fraction by any number does not change the value of the fraction.*

DEMONSTRATION OF THE FIRST.—If we multiply the numerator of a fraction by any number, as 5, the resulting fraction will express 5 times as many parts each of the same size as before; hence the valuation of the fraction is 5 times as great.

DEMONSTRATION OF THE THIRD.—Since the denominator shows the number of equal parts into which the unit is divided, if we multiply the denominator by any number, as 4, the unit will be divided into 4 times as many parts; hence each part will be $\frac{1}{4}$ as large as before, and the same number of parts being taken, the value of the fraction will be $\frac{1}{4}$ as great.

DEMONSTRATION OF THE FIFTH.—Since multiplying the numerator multiplies the value, and multiplying the denominator divides the value, of the fraction, multiplying both numerator and denominator by the same number multiplies and divides the value by the same number, and hence does not change its value. Therefore, etc.

NOTE.—The 2d is demonstrated very much like the 1st, the 4th like the third, the 6th like the 5th.

SECTION IV.

DENOMINATE NUMBERS.

A DENOMINATE NUMBER is a number in which the unit is a measure.

A MEASURE is a unit by which quantity of magnitude is estimated numerically.

Denominate numbers are of eight classes, namely:

1. Value,
2. Weight,
3. Length,
4. Surface,
5. Volume,
6. Capacity,
7. Time,
8. Circular.

NOTE.—Nature, regarded as *how many* and *how much*, gives rise to two kinds of quantity, quantity of *multitude* and quantity of *magnitude*. Quantity of multitude exists in units; in quantity of magnitude, the unit must be assumed in order to express it numerically. The numerical expression of quantity of magnitude gives rise to denominate numbers.

LESSON I.

Measures of Value.

MONEY is the measure of the value of things. It is of two kinds, *coin* and *paper money*.

UNITED STATES MONEY.

UNITED STATES MONEY is the legal currency of the United States. It is also called *Federal Money*.

TABLE.

10 mills (*m.*)	equal	1 cent, . . .	*ct.*
10 cents	"	1 dime, . . .	*d.*
10 dimes	"	1 dollar, . . .	$.
10 dollars	"	1 eagle, . . .	*E.*

NOTES.—1. Federal money was adopted in 1786. The cent was proposed by Robert Morris, and named by Thomas Jefferson.

2. Shillings and pence were used long after the adoption of Federal money. In New England $1 equaled 6 shillings; in New York and New Jersey, 8 shillings; in Pennsylvania, 7 shillings 6 pence, etc.

3. The word *dollar* is supposed to be derived from the German *Dale*, a town; the term *dime* is from the French *disme*, meaning ten; the term *cent* is from the Latin *centum*, a *hundred;* the term *mill* is from the Latin *mille*, a *thousand.*

1. How many mills in 4 cents? in 7 cents? in 8 dimes?

2. How many cents in 5 dimes? in 6 dollars? in 3 eagles?

3. How many dollars in 9 eagles? in 40 dimes? in 500 cents?

4. How many cents in $\$\frac{1}{2}$? $\$\frac{1}{4}$? $\$\frac{3}{4}$? $\$\frac{1}{8}$? $\$\frac{3}{8}$? $\$\frac{5}{8}$?

5. What part of $1 is 10 cts.? $12\frac{1}{2}$ cts.? 20 cts.? 25 cts.? $16\frac{2}{3}$ cts.? $33\frac{1}{3}$ cts.? $37\frac{1}{2}$ cts.? 50 cts.? $62\frac{1}{2}$ cts.? 75 cts.? $83\frac{1}{3}$ cts.?

6. What part of 4 eagles is 8 dimes, and what part of 8 cents is $\frac{2}{5}$ of a dime?

7. How many eagles in 50 dollars? in 300 dimes? in 7000 cents?

ENGLISH OR STERLING MONEY.

ENGLISH or STERLING MONEY is the currency of England.

TABLE.

4 farthings (*qr.*)	equal 1 penny, . . .	*d.*
12 pence	" 1 shilling, . . .	*s.*
20 shillings	" 1 pound, . . .	£.
21 shillings	" 1 guinea, . . .	*g.*

1 £ = $4.84. 1 *s.* = $0.24. 1 *d.* = $0.02.

NOTES.—1. *Sterling* is derived from *Easterling*, the name of the early traders who came from the East to England. The £, coined in gold, is called a *sovereign.* A five-shilling piece in silver is called a *crown.*

2. *Farthing* is from "*four things*," the old English penny being marked with a cross, so that it could be broken into four parts, called *four things.* The *guinea* is so called because the gold of which it was first made came from Guinea in Africa.

3. The symbols £, *s.*, *d.*, and *qr.* are the initials of the Latin words *libra*, *solidus*, *denarius*, and *quadrans*, signifying pounds, shillings, penny, and quarter.

1. How many farthings in 2 pence? in 3? in 5? in 6? in 8? in 10?

2. How many pence in 8 farthings? in 2? in 20? in 28? in 30?

3. How many pence in 2 shillings? in 4? in 5? in 6? in 10?

4. How many shillings in 24 pence? in 48? in 72? in 96? in 150?

5. What part of 2 pence is 6 farthings, and what part of 3 shillings is 5 pence?

6. What part of 16 pence is $\frac{2}{3}$ of a shilling, and what part of a guinea is $\frac{3}{4}$ of a pound?

LESSON II.

Measures of Weight.

MEASURES OF WEIGHT are measures used to determine the force by which bodies are drawn toward the earth by gravity.

Measures of weight are of three kinds: *Troy Weight*, *Apothecaries' Weight*, and *Avoirdupois Weight*.

Troy Weight.

Troy Weight is used for weighing gold, silver, precious stones, and in philosophical experiments.

TABLE.

24 grains (*gr.*)	equal	1 pennyweight, .	*pwt.*
20 pennyweights . . .	"	1 ounce, ...	*oz.*
12 ounces	"	1 pound, ...	*lb.*

1. The term *Troy* is said to be derived from *Troyes*, the name of a town in France, where the weight was first used in Europe. The symbol (*oz.*) is from the Spanish word *onza*, for ounces, and (*lb.*) from *libra*, a pound.

2. Diamonds are weighed by *carats* and parts of carats. A carat equals 4 Troy grains. The purity of gold is also expressed in carats—a carat meaning $\frac{1}{24}$ part.

1. How many grains in 2 pwt.? in 3 pwt.? in 4 pwt.? in 5 pwt.? in 6 pwt.?

2. How many pwts. in 48 gr.? in 72 gr.? in 96 gr.? in 120 gr.? in 240 gr.?

3. How many pwts. in 3 oz.? in 4 oz.? in 5 oz.? in 6 oz.? in 7 oz.? in 10 oz.?

4. How many ounces in 5 pounds? in 7 lb.? in 10 lb.? in 8 lb.? in 4 lb.? in 6 lb.?

5. If 10 pwts. of silver are worth 3 shillings, what is the value of 3 lbs. of silver?

6. What is the value of 3 ounces of gold, if 3 pwt. are worth 3 dollars?

Apothecaries' Weight.

Apothecaries' Weight is used in prescribing and mixing medicines.

TABLE.

20 grains (*gr.*)	equal	1 scruple, . .	℈
3 scruples	"	1 dram, . . .	ʒ
8 drams	"	1 ounce, . .	℥
12 ounces	"	1 pound, . .	℔

The pound is the same as the pound Troy. Medicines are bought and sold in quantities by Avoirdupois Weight.

1. How many grs. in 3 ℈? in 5 ʒ? in 2 ℥? in 1 ℔.?

2. How many scruples in 4 ʒ? in 40 gr.? in 2 ℥ and 3 ʒ?

3. How many ounces in 3 ℔.? in 16 ʒ? in 4 ℔. and 5 ℥?

4. How many drams in 120 gr.? in 36 ℈? in 3 pounds?

5. If 5 grs. of medicine cost 10 cents, what will 3 ʒ and 4 ℈ cost?

6. Two-thirds of 9 scruples of a certain drug cost 18 cents; what will 3 fourths of 8 pounds cost?

Avoirdupois Weight.

Avoirdupois Weight is used for weighing everything except jewels and the precious metals.

TABLE.

16 drams (*dr.*)	equal	1 ounce, ...	*oz.*
16 ounces	"	1 pound, ...	*lb.*
25 pounds	"	1 quarter, ...	*qr.*
4 quarters	"	1 hundred weight,	*cwt.*
20 hundred weight . .	"	1 ton, ...	*T.*

1. The term *Avoirdupois* is derived from the French *avoir du poids*, signifying *to have weight.*

2. The pound consists of 7000 Troy grains. A *Cental* (100 lbs.) is a new measure used in buying and selling grain.

1. How many drams in 2 oz.? in 3 oz.? in 5 oz.? in 10 oz.?

2. How many ounces in 3 pounds? in 5 lb.? in 48 drams?

3. How many quarters in 75 pounds? in 100 lb.? in 5 cwt.?

4. How many pounds in 2 qr.? in 1 cwt.? in 1 ton? in 64 oz.?

5. How many hundred weight in 36 qr.? in 300 lb.? in 6 tons?

6. What will 5 pounds of starch cost, if 5 ounces cost $3\frac{3}{4}$ cents?

7. What will 2 cwt. of coffee cost, at the rate of 4 pounds for 60 cents?

8. I gave 3 cwt. 2 qr. of hay, worth $20 a ton, for butter worth 25 cents a pound; how many pounds of butter did I receive?

9. Which is the heavier, a pound of gold or a pound of lead? An ounce of silver or an ounce of feathers?

LESSON III.

Measures of Extension.

MEASURES OF EXTENSION are measures used to determine the *length*, *surface*, and *volume* of bodies.

Measures of Extension are of three kinds: *Long Measure*, *Surface Measure*, and *Cubic Measure*.

Long Measure.

Long Measure is used in measuring lengths, distances, etc. It is sometimes called *Linear Measure*.

Table.

12 inches (*in.*)	equal	1 foot, . . .	*ft.*
3 feet	"	1 yard, . . .	*yd.*
5½ yards	"	1 rod, . . .	*rd.*
40 rods	"	1 furlong, . .	*fur.*
8 furlongs	"	1 mile, . . .	*m.*

1. The old table of *cloth measure* is now seldom used, cloth being measured by the yard, half yard, etc.

2. Besides the measures in the table, we have the *hand* (4 in.), used in measuring the height of horses; the *pace* (3 ft.), used in measuring distances; the *fathom* (6 ft.), used in measuring the depth of water; the *league* (3 miles), used in measuring distances at sea.

1. How many inches in 3 feet? in 5 ft.? in 7 ft.? in 2 yd.? in 2 rods and 2 feet?

2. How many feet in 60 inches? in 96 in.? in 108 in.? in 6 rd.? in 4 rd. 7 yd.?

3. How many yards in 45 feet? in 66 ft.? in 72 in.? in 4 rd.? in 1 fur. 4 rd.?

4. How many furlongs in 320 rods? in 440 rods? in 9 miles?

5. How many miles in 104 furlongs? in 640 fur.? in 1760 yds.?

6. What part of 1 yard is 2 feet, and what part of 2 furlongs is 5½ yards?

Surface or Square Measure.

Surface or Square Measure is used in measuring surfaces, as land, boards, etc.

TABLE.

144 square inches (*sq. in.*) .	equal	1 square foot, *sq. ft.*
9 square feet	"	1 square yard, *sq. yd.*
30¼ square yards.	"	1 perch, . . *P.*
40 perches	"	1 rood, . . . *R.*
4 roods	"	1 acre, . . . *A.*
640 acres	"	1 square mile, *sq. m.*

The *perch* is the same as a *square rod.* The *rood* is less used than formerly. In surveying, we use Gunter's chain, which is 4 rods or 66 feet long, and consists of 100 links. A *square chain* equals 16 sq. rd.; 10 *square chains* equal 1 acre.

1. How many square feet in 6 square yards? in 8 sq. yd.? in 10 sq. yd.?

2. How many square yards in 36 square feet? in 63 sq. ft.? in 108 sq. ft.?

3. How many perches in 3 roods? in 4 roods? in 60½ sq. yd.? in 121 sq. yd.?

4. How many roods in 80 perches? in 120 perches? in 3 acres? in 5 acres?

5. What is the difference between 3 feet square and 3 square feet? 4 inches square and 4 square inches?

Cubic Measure.

Cubic or Solid Measure is used in measuring things which have length, breadth, and thickness.

TABLE.

1728 cubic inches (*cu. in.*) . .	equal	1 cubic foot, *cu. ft.*
27 cubic feet.	"	1 cubic yard, *cu. yd.*
16 cubic feet.	"	1 cord foot, *cd. ft.*
8 cord feet or 128 cubic feet	"	1 cord of wood, *Cd.*

1. A cubic yard of earth is called a *load*, and 24¾ cubic feet of stone or masonry make a *perch.*

2. A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains one cord, and one foot in length of such a pile is one *cord foot.*

LESSON IV.

Measures of Capacity.

MEASURES OF CAPACITY are volumes used to determine the quantity of fluids and many dry substances.

Measures of Capacity are of two general kinds: *Liquid Measure* and *Dry Measure.*

Liquid Measure.

Liquid Measure is used in measuring nearly all kinds of liquids. It is often called *Wine Measure.*

TABLE.

4 gills (*gi.*)	equal	1 pint, . . .	*pt.*
2 pints	"	1 quart, . . .	*qt.*
4 quarts	"	1 gallon, . . .	*gal.*

1. The standard liquid gallon contains 231 cubic inches. Beer, ale, and milk were formerly sold by *Beer Measure*, the gallon of which consists of 282 cubic inches. Beer is still occasionally sold by this measure.

2. In the old tables were given 31½ gallons = 1 barrel; 63 gallons = 1 hogshead; 2 hogsheads = 1 pipe; 2 pipes = 1 tun. These are not exact measures, but vessels of variable capacity.

1. How many gills in 6 pints? in 5 quarts? in 3 gallons? in 5 gallons?

2. How many pints in 24 gills? in 8 quarts? in 2 gallons? in 4 gallons?

3. How many quarts in 40 gills? in 24 pints? in 4 gallons? in 2 gallons and 2 quarts?

4. What part of 2 quarts is 2 gills? What part of 3 gallons is 6 pints?

5. A exchanged 8 quarts of wine, at $2 a pint, for beer worth 50 cents a gallon; how much beer did he get?

Dry Measure.

Dry Measure is used in weighing dry substances, as grain, fruit, salt, coal, etc.

TABLE.

2 pints (*pt.*)	equal	1 quart, . . .	*qt.*
8 quarts	"	1 peck, . . .	*pk.*
4 pecks	"	1 bushel, . . .	*bu.*

1. The unit of dry measure is the *bushel*, which is a cylinder 18½ inches in diameter and 8 inches deep. One-half of a peck, or 4 quarts, is sometimes called a *dry gallon*.

2. The measure must be *even* full in measuring grain, seeds, etc.; but in measuring apples, potatoes, corn on the ear, etc., it must be *heaping* full. 4 heaped pecks equal 5 even pecks; 6 dry quarts equal nearly 7 liquid quarts.

1. How many pints in 4 quarts? in 3 pecks? in 2 bushels?

2. How many quarts in 6 pecks? in 5 bushels? in 24 pints?

3. How many pecks in 96 pints? in 56 quarts? in 5½ bushels 3 pecks?

4. How many bushels in 64 quarts? in 128 pints? in 48 pecks?

5. At ten cents a peck, how many bushels of apples can be bought for 8 dollars?

6. Which cost the most, and how much, 5 bushels 3 quarts of salt, at 4 cents a quart, or 10 bushels 3 pecks of apples, at 50 cents a bushel?

LESSON V.

Measures of Time and Circles.

Measure of Time.

TIME MEASURE is the measure of time or duration.

TABLE.

60 seconds (*sec.*)	equal	1 minute, . .	*m.*
60 minutes	"	1 hour, . . .	*hr.*
24 hours	"	1 day, . . .	*da.*
365 days	"	1 year, . . .	*yr.*
100 years	"	1 century, .	*C.*

1. Also, 7 days = 1 week; 4 weeks = 1 lunar month; 13 lunar months, 1 day, and 6 hours = 1 year; 12 calendar months = 1 year. Every fourth year contains 366 days, and is called *leap year*.

2. The year is divided into 4 *seasons* of three months each. Seven months contain 31 days each; four contain 30 days each; and one, February, contains 28 days every common year, and 29 days in leap year. The months which contain 30 days each may be remembered by the following extract:

Thirty days hath September,
April, June, and November.

1. How many seconds in 2 minutes? in 3 min.? in 6 min.?

2. How many minutes in 3 hours? in 4 hours? in 120 seconds?

3. How many hours in 2 days? in 3 days? in 240 minutes?

4. How many days in 3 weeks? in 8 weeks? in 48 hours?

5. Name the months which have 30 days each. Those having 31 days each. How many days has February?

Circular Measure.

Circular Measure is used to measure arcs of circles, angles, etc.

TABLE.

60 seconds (″)	equal	1 minute, . . . ′
60 minutes	"	1 degree, . . . °
30 degrees	"	1 sign, ... *S.*
12 signs or 360°	"	1 circumference, *C.*

Miscellaneous.

12 units = 1 dozen.	20 units = 1 score.
12 dozen = 1 gross.	24 sheets = 1 quire.
12 gross = 1 great gross.	20 quires = 1 ream.

The Metric System.

The Metric System of weights and measures is now used in many countries, and has been legalized in the United States. The principal units of this system are as follows:

1. *Weight.*—The *Unit of Weight* is the *Gram.* The *gram* = $15\frac{44}{100}$ Troy grains; it is used for light weights. The *kilogram* (1000 grains), called also *kilo*, is used for ordinary weights. The *kilogram* = $2\frac{1}{2}$ lbs. Avoirdupois, nearly.

2. *Length.*—The *Unit of Length* is the *Meter.* It equals $39\frac{37}{100}$ inches, or very nearly 3 feet 3 inches and $\frac{3}{8}$ of an inch. Long distances are measured by the *kilometer* (1000 *meters*), which equals nearly $\frac{5}{8}$ of a mile.

3. *Surface.*—The *Unit of Surface* is the *Square Meter.* Land is measured by the *Are*, or *square dekameter* (square of 10 meters). One *acre* equals very nearly 40 *ares.*

4. *Volume.*—The *Unit of Volume* is the *Cubic Meter*, called the *Stere.* It equals about $35\frac{1}{3}$ cubic feet.

5. *Capacity.*—The *Unit of Capacity* is the *Liter*, equal to $1\frac{1}{18}$ liquid quarts, very nearly. The *hectoliter* (100 *liters*), equal to about $2\frac{5}{8}$ bushels, is used for measuring grain, etc.

TABLE OF PRICES.

Per doz.	Each.	Per doz.	Each.	Per doz.	Each.	Per doz.	Each.
.25	.02+	3.25	.27+	6.25	.52+	9.25	.77+
.50	.04+	3.50	.29+	6.50	.54+	9.50	.79+
.75	.06+	3.75	.31+	6.75	.56+	9.75	.81+
1.00	.08+	4.00	.33+	7.00	.58+	10.00	.83+
1.25	.10+	4.25	.35+	7.25	.60+	10.25	.85+
1.50	.12½	4.50	.37½	7.50	.62½	10.50	.87½
1.75	.15—	4.75	.40—	7.75	.65—	10.75	.90—
2.00	.17—	5.00	.42—	8.00	.67—	11.00	.92—
2.25	.19—	5.25	.44—	8.25	.69—	11.25	.94—
2.50	.21—	5.50	.46—	8.50	.71—	11.50	.96—
2.75	.23—	5.75	.48—	8.75	.73—	11.75	.98—
3.00	.25	6.00	.50	9.00	.75	12.00	1.00

Note.—Pupils will commit the above table to memory, reciting it at first by naming price per dozen and price each, and then *vice versa.*

This table is of great practical utility in all business transactions of buying or selling by the dozen or fixing retail prices.

LESSON VI.

Miscellaneous Problems.

HOW much will 9 eggs cost, at 20 cents a dozen?

2. How much will 8 pearl buttons cost, at 75 cents a dozen?

3. What will 4 table-spoons cost, at the rate of $11.52 a gross?

4. What will shoe-tacks cost a dozen, at the rate of $2.88 a great gross?

5. What will 8 sheets of paper cost, at 18 cents a quire?

6. What will 12 quires of paper cost, at $4 a ream?

7. What cost ½ a gross of lead-pencils, at 50 cents a dozen?

8. What will 2 reams of sand-paper cost, at 22 cents a quire?

9. If a ream of emery-paper costs $12, what does it cost a quire?

10. What will 4 gross of pens cost, if the wholesale rate is 2 pens for a cent?

11. A vessel was sunk in 8 fathoms of water; how many feet deep was it?

12. How many feet high is a horse which measures 15½ hands high?

13. How much will 3 pecks of potatoes cost, at 80 cents a bushel?

14. When apples sell at 20 cents a peck, how much are they worth a bushel?

15. What will 3 quarts of molasses cost, at 96 cents a gallon?

16. How many vials, holding a gill each, can be filled with half a gallon of alcohol?

17. How many half-pint bottles will 2 gallons of Arnold's writing-fluid fill?

18. If I pay $1.60 for a bushel of blackberries, how much is that a quart?

19. If it takes 6 buttons for one shirt, how many shirts will 2 gross of buttons trim?

20. The distance across a city park is 132 paces; how many rods is it?

21. What will 15 pounds of flour cost, at the rate of $4 a hundred weight?

22. A boy bought a bushel of chestnuts for $2.40, and sold them at 12 cents a quart; what did he gain?

23. I bought 30 lbs. of flour for $1.50; how much is that a hundred weight?

24. Albert's horse is 5 feet in height; how many hands high is his horse?

25. A vessel sails 4 leagues an hour; how many hours will it take to sail 120 miles?

26. Sarah picked a peck of cherries, and sold them at 4 cents a pint; how much did she receive for them?

27. A sailor took the sounding, and found the water 50 feet deep; how many fathoms was it?

28. My grandfather's age is "three score years and ten;" how old is he?

29. A merchant buys $\frac{1}{4}$ dozen hand-saws, at $20 a dozen; how shall he sell them to gain 50 cents apiece?

30. I paid $3.20 for a bushel of huckleberries; how shall I sell them a quart to gain 2 cents a quart?

31. A merchant bought $\frac{1}{2}$ dozen shovels, at $9.60 a dozen; how shall he retail them to gain 20 cents apiece?

32. What must I pay a gross for hair-pins, that I may sell them for five cents a dozen, and gain 2 cents a dozen?

33. Bought paper collars at 40 cents a box, each box containing a dozen; how much will I gain on each collar by selling them for 5 cents apiece?

34. A grocer paid $6.40 for a bushel of cranberries; how shall he sell them a quart to gain 2 cents a pint?

SECTION V.

PROPORTION.

LESSON I.

Given the Sum of the Parts.

HARRY'S age, increased by $\frac{1}{2}$ of his age, equals 24 years; what is his age?

Solution.—By the condition of the problem, $\frac{2}{2}$ of Harry's age, plus $\frac{1}{2}$ of his age, which is $\frac{3}{2}$ of his age, equals 24 years. If $\frac{3}{2}$ of Harry's age equals 24 years, $\frac{1}{2}$ of his age equals $\frac{1}{3}$ of 24 years, which is 8 years, and $\frac{2}{2}$ of his age equals 2 times 8, or 16 years. Therefore, etc.

2. What number is that to which, if its $\frac{1}{3}$ be added, the sum will be 36?

3. Required the number which, being increased by its $\frac{2}{3}$, equals 40.

4. What number is that which, being increased by its $\frac{3}{7}$, the sum will be 80?

5. What number is that which, being increased by its $\frac{3}{5}$, the sum will be 40?

6. Three times a certain number, increased by $\frac{2}{5}$ of itself, equals 22; required the number.

7. Reuben's age, being doubled and increased by $\frac{3}{4}$ of his age, equals 55 years; how old is he?

8. Three and $\frac{1}{2}$ times a number, plus $\frac{2}{3}$ of the number, equals 50; what is the number?

9. Two-fifths of a number, being increased by $\frac{1}{2}$ of the number, equals 27; required the number.

10. What number is that which, being increased by the difference between its $\frac{1}{4}$ and $\frac{1}{5}$, equals 42?

11. A boy being asked his age, replied that his age, increased by its $\frac{1}{2}$ and $\frac{2}{5}$, equaled 39 years; what was his age?

12. A, having $\frac{2}{5}$ of a dollar, gave $\frac{1}{2}$ of it to B, and B gave $\frac{1}{4}$ of his to C; what part of a dollar did each then have?

13. When B was married he was 25 years old, and $\frac{3}{5}$ of his age was 3 years more than $\frac{2}{3}$ of his wife's age; required the age of his wife.

14. The distance from Medway to Columbia is 42 miles, and $\frac{4}{7}$ of this distance is $\frac{1}{3}$ of the distance from Medway to Rockville; required the distance.

15. A fishing-rod is 16 feet long, and $\frac{3}{4}$ of its length lacks 2 feet of being $\frac{2}{3}$ of the length of the line; required the length of the line.

16. A watch cost \$40, and this is $\frac{2}{3}$ of $\frac{3}{4}$ of the cost of the watch and chain together; required the cost of the chain.

17. Susan has 7 peaches, and $\frac{4}{5}$ of Susan's number, minus $\frac{3}{5}$ of a peach, is $\frac{5}{8}$ of Elizabeth's number; how many has Elizabeth?

18. If there are 50 chestnuts in a pint, how many do A and B receive respectively, if A has $\frac{4}{5}$ of a pint, and B has $\frac{3}{4}$ as many as A?

19. A has 60 fruit trees, $\frac{2}{5}$ of which bear peaches, $\frac{2}{3}$ of the remainder pears, and the remainder apples; how many are there of each?

20. Benton lost $\frac{4}{5}$ of all his money, and then found $\frac{5}{4}$ as much as he lost, and then had \$120; how much money had he at first?

21. Mary gave $\frac{3}{4}$ of her money to the poor, and then found $\frac{2}{3}$ as much as she gave away, and then had \$30; how much had she at first?

22. William borrowed $\frac{2}{3}$ of Emily's money, and after spending $\frac{3}{4}$ of it returned the remainder, which was \$20; how much money had Emily?

LESSON II.

Given the Difference of the Parts.

A MAN spent $\frac{3}{5}$ of his money for a horse, and then had $60 remaining; how much money had he at first?

Solution.—After spending $\frac{3}{5}$ of his money, there remained $\frac{5}{5} - \frac{3}{5}$, or $\frac{2}{5}$ of his money, which equals $60. If $\frac{2}{5}$ of his money equals $60, $\frac{1}{5}$ of his money equals $\frac{1}{2}$ of $60, which is $30, etc.

2. A lady, after giving away $\frac{1}{3}$ of her money, had only 40 cents remaining; how much money had she at first?

3. A farmer sold $\frac{3}{5}$ of his cows, and then had 25 cows remaining; how many had he at first?

4. Mary gave $\frac{1}{2}$ of her money for silk and $\frac{1}{3}$ for satin, and had $10 left; how much had she at first?

5. Henry's money, diminished by its $\frac{1}{5}$ and $\frac{1}{6}$, equals 57 dollars; how much money has he?

6. Mr. A's money, increased by its $\frac{1}{2}$ and $\frac{1}{6}$, equals 30 dollars; how much money has he?

7. Says B to C, $\frac{3}{5}$ of my age, diminished by $\frac{2}{6}$ of it, equals 24 years; how old was he?

8. Peter gave 10 cents for a pie, which is $\frac{3}{4}$ of $\frac{1}{2}$ of the cost of his supper; required the cost of his supper.

9. Mr. K's hat cost $6, which was $3 less than $\frac{3}{5}$ of the cost of his coat; required the cost of the coat.

10. What number is that which, being doubled and then diminished by its $\frac{3}{4}$, equals 60?

11. A boy lost 4 marbles and found 10, and then had $\frac{3}{2}$ as many as at first; how many had he at first?

12. $\frac{1}{3}$ of the length of a pole is in the air, $\frac{1}{4}$ in the water, and 10 feet in the ground; required the length of the pole.

13. A lady, being asked her age, replied that her daughter's age is 8 years, which is $\frac{4}{5}$ of $\frac{1}{4}$ of her age; required her age.

14. Frank, after spending $\frac{2}{3}$ of his money, found that $16 was $\frac{2}{3}$ of what he had remaining; how much money had he at first?

15. If $\frac{2}{5}$ of an army were killed, $\frac{2}{5}$ taken prisoners, and 800 men escaped, of how many men did the army consist?

16. When E was married, he was 27 years old, and $\frac{2}{3}$ of his age was 4 years more than $\frac{2}{3}$ of his wife's age; required the age of his wife.

17. A boy, after spending $\frac{1}{4}$ of his money for candies and $\frac{1}{8}$ for peaches, found that 20 cents was $\frac{2}{7}$ of what remained; how much money had he?

18. What number is that which, being increased by its $\frac{1}{2}$, and that sum diminished by $\frac{2}{3}$ of the number, the remainder is 50?

19. A thief stole $\frac{3}{5}$ of Harry's money, and before he was caught spent $\frac{2}{3}$ of it; the remainder, which was $20 less than he stole, was given back; how much money had Harry?

20. A kite in the air fell $\frac{1}{2}$ of the distance to the ground, then arose $\frac{1}{3}$ of the distance it was from the ground, and then fell $\frac{1}{4}$ of the distance it arose; what part of the whole distance was it from the ground?

LESSON III.

A Part Increased or Diminished.

MARTHA'S age, increased by 6 years, equals 20 years; how old is Martha?

Solution.—If Martha's age, increased by 6 years, equals 20 years, Martha's age is 20 years minus 6 years, or 14 years.

2. What number is that which, being increased by 12, equals 26?

3. Two-thirds of A's money, increased by $8, equals $32; how much money has A?

4. Sarah has 25 roses, which equals 4 more than $\frac{1}{2}$ of Mary's number; how many has Mary?

5. A man earned a certain sum of money, and then found $10, and then had $48; how much did he earn?

6. Henry's age, diminished by 6 years, equals 14 years; how old is Henry?

7. How heavy is Sarah, if her weight, diminished by 20 pounds, equals 60 pounds?

8. What is the height of a tree, if 40 feet equals the height diminished by 20 feet?

9. Required A's money, if twice his money, diminished by $12, equals $38.

10. What is B's age, if $\frac{3}{4}$ of his age, increased by 8 years, equals 35 years?

11. A certain number, increased by 4 times itself and 15, equals 55; what is the number?

12. A sum of money, diminished by $\frac{3}{7}$ of itself and $6, equals $12; what is the sum?

13. The cost of a horse, diminished by $\frac{1}{3}$ of the cost and $20, equals $80; what did the horse cost?

14. A fishing-pole which is 10 feet long equals $\frac{3}{4}$ of the length of the line, minus 2 feet; required the length of the line.

15. Having a farm, I sold $\frac{1}{3}$ of it, and afterward bought 24 acres, and then had 56 acres; how much land had I at first?

16. Mary spent $\frac{1}{4}$ of her money for ribbons and 25 cents for roses, and had 50 cents left; what was her money?

17. James, having twice as much money as John, lost $\frac{1}{3}$ of it, then earned $15, and then had $45; how much money had each?

18. If twice my weight be diminished by $\frac{1}{3}$ of my weight and 30 pounds, it will equal 180 pounds; required my weight?

19. Mary culled 3 times as many roses as Jane; she

sold $\frac{1}{6}$ of them, and culled 10 more, and then had 30; how many did each cull?

20. A watch and chain cost \$150, and $\frac{2}{3}$ of the cost of the watch, plus \$15, equals \$95; required the cost of each.

21. A fishing-rod is 15 feet long, and $\frac{3}{5}$ of its length lacks 3 feet of being $\frac{6}{7}$ of the length of the line; required the length of the line.

22. A tree is 60 feet high, which is $\frac{5}{6}$ of $\frac{6}{7}$ of the length of its shadow, diminished by 20 feet; required the length of the shadow.

LESSON IV.

One Part More or Less than Another.

IF two times a number, increased by 8, equals 3 times the number, what is the number?

Solution.—If 2 times a number, increased by 8, equals 3 times the number, 8 must be the difference between 3 times the number and 2 times the number, which is once the number. Hence, the number is 8.

2. A's money, increased by \$18, equals 3 times his money; what is his money?

3. B's age, increased by 42 years, equals 4 times his age; what is B's age?

4. Three times the length of a pole, diminished by 20 feet, equals once the length; required the length of the pole.

5. If 4 times a number, diminished by 8, equals 2 times the number, what is the number?

6. Two-thirds of Mary's number of roses, diminished by 10, equals $\frac{1}{4}$ of her number; how many roses has she?

7. Barton lost 6 cents more than $\frac{1}{3}$ of his money, and then had $\frac{2}{5}$ of his money; what was his money?

8. A storm broke off 10 feet less than $\frac{1}{3}$ of a tree, and there remained $\frac{3}{4}$ of the tree; required the length of the tree.

9. Three times the number of sheep a farmer has equals $2\frac{1}{2}$ times the number, increased by 8; how many sheep has he?

10. Mary spent 12 cents more than $\frac{1}{2}$ of her money for a book, and then had remaining $\frac{1}{3}$ of her money; what was her money?

11. Three times a number, plus 4, equals 2 times the same number, plus 12; what is the number?

Solution.—If 3 times a number, plus 4, equals 2 times the number, plus 12, 3 times the number equals 2 times the number, increased by the difference between 12 and 4, or 8, etc.

12. Three times a number, plus 2, equals 2 times the number, plus 6; what is the number?

13. Three times a number, plus 8, equals 4 times the same number, plus 3; required the number.

14. One-half of a number, increased by 10, equals $\frac{2}{3}$ of the same number, plus 8; what is the number?

15. Four times Edgar's age, diminished by 10 years, equals 3 times his age, increased by 10 years; what is his age?

16. If the height of a tree be increased by its $\frac{2}{3}$ and 10 feet more, the sum will be twice the height; what is the height of the tree?

17. If twice the length of a pole be increased by its $\frac{3}{5}$ and 2 feet more, the sum will equal 3 times the length of the pole; required its length.

18. If 3 times Harry's age be increased by its $\frac{5}{6}$, and 2 years more, the sum will equal 4 times his age; what is his age?

19. Two-thirds of Morton's apples, increased by 2, equals $\frac{3}{4}$ of his number, diminished by 1; how many apples has he?

20. If the height of a steeple be increased by its $\frac{1}{2}$,

and that sum diminished by the difference between $\frac{1}{4}$ and $\frac{1}{5}$ of the sum, it will equal $\frac{7}{4}$ of its height, minus 13 feet; required its height.

21. Baldwin had stolen from him $\frac{5}{7}$ of his money, and the thief was not caught until he had spent $\frac{4}{5}$ of it; the remainder, which was $30 less than Baldwin had remaining, was given back; how much money had Baldwin?

LESSON V.

One Part a Number of Times Another.

WILLIAM and Henry have 15 marbles; how many has each, provided William has twice as many as Henry?

Solution.—By a condition of the problem, twice Henry's number equals William's, which, added to Henry's number, equals three times Henry's, which is what they both have, or 15 marbles. If 3 times Henry's number equals 15, once his number equals $\frac{1}{3}$ of 15, which is 5, and twice his number, or William's number, equals twice 5, or 10 marbles. Therefore, etc.

2. Robert has 3 times as many cents as Elias, and they together have 24; how many has each?

3. William has 4 times as many nuts as Oliver, and they together have 20 pints; how many pints has each?

4. Divide the number 25 into two such parts that 4 times one part shall equal the other.

5. The sum of two numbers equals 40, and $\frac{1}{3}$ of the greater equals the less; required the numbers.

6. Emma has 35 flowers, and 4 times the number of roses equals the number of pinks; how many has she of each kind?

7. A father and son earned in one week $12; how much did each earn, if the father earned twice as much as the son?

8. A pole, 36 feet in length, was broken into two unequal pieces, such that $\frac{1}{3}$ of the longer piece equals the shorter; required the length of each piece.

9. In a certain school, consisting of 35 scholars, there were $\frac{1}{4}$ as many girls as boys; how many boys and how many girls in the school?

10. A man bought a horse and cow for $100, and the cow cost $\frac{2}{3}$ as much as the horse; required the cost of each.

11. A watch and chain cost 42 dollars; what was the cost of each, provided $\frac{2}{5}$ of the cost of the watch equals the cost of the chain?

12. Harry and Thomas lost a purse of money containing $24, of which Harry owned $\frac{5}{7}$ as much as Thomas; how much did each lose?

13. Marie has 40 cherries more than Jane, and 5 times Jane's number equals Marie's; how many cherries has each?

14. Two-thirds of 30 is $\frac{5}{2}$ of the difference between two numbers, and the less is $\frac{3}{5}$ of the greater; what are the numbers?

15. Divide 36 apples among three boys, so that the second may have twice as many as the first, and the third 3 times as many as the first.

16. Divide 66 plums among Ella, Emma, and Ettie, so that Ella shall have twice and Emma three times as many as Ettie.

17. A, B, and C, together, earned $70; A earned twice as much as B, and B twice as much as C; how much did each earn?

18. The sum of three numbers is 50; the second is 3 times the first, and the third is twice the second; what are the numbers?

19. A turkey, duck, and hen cost 22 dimes; the duck cost twice as much as the hen, and the turkey cost 4 times as much as the duck; required the cost of each.

20. A man bought a horse, cow, and sheep for $105; how much did he pay for each, provided the cow cost 4 times as much as the sheep, and the horse 4 times as much as the cow?

21. Of a certain pole whose parts are in the mud, air, and water, $\frac{2}{3}$ of the length in the air equals the length in the water, and $\frac{3}{4}$ of the length in the water equals the length in the mud; required the length of each part, supposing the part in the water to be 10 feet longer than the part in the mud.

LESSON VI.

One Part a Given Number More than Another.

A AND B have 25 oranges; how many has each, if B has 5 more than A?

Solution.—By a condition of the problem, A's number + 5 oranges equals B's number, which, added to A's, is twice A's number + 5, which equals 25 oranges. If twice A's + 5 = 25, twice A's = 25 − 5, or 20; if twice A's = 20, once A's equals $\frac{1}{2}$ of 20, which is 10; and since B had 5 more than A, 10 + 5, or 15, equals B's number. Therefore, etc.

2. Mary has seven oranges more than William, and they together have 27; how many has each?

3. Stephen has 10 cents more than Martha, and they together have 40; how many has each?

4. The sum of two numbers is 31, and their difference 5; what are the numbers?

5. Divide the number 28 into two such parts, that one part may be 6 less than the other.

6. Thomas and Reuben each earned the same sum of money, when Reuben found $9, and they then together had $45; how much did each earn?

7. Ella and Kate had each the same number of candies; Ella ate 5 of hers, and they then together had 21; how many had each at first?

8. Two boys found an equal number of cents; one lost 6 and the other 4, and they then together had 22; how many did each find?

9. A and B had equal sums of money; A lost $5, and B earned $7, and they then together had $36; how much had each at first?

10. Daniel and Edwin had each the same number of peaches; Daniel lost 6, and Edwin gave him 4, and they then together had 14; how many had each then?

11. Three times Harry's age, increased by 5 years, equals Harvey's age, and the sum of their ages is 45 years; how old is each?

12. Divide the number 48 into two such parts, that twice the first part, diminished by 6, shall equal the second part.

13. The sum of two numbers is 55, and the greater equals 3 times the less, diminished by 5; required the numbers.

14. A pole, whose length was 48 feet, was broken into two unequal pieces, $\frac{2}{5}$ of the longer part equaling the shorter; required the length of each piece.

15. A watch and chain cost $85, and $\frac{3}{10}$ of the cost of the watch, plus $7, equals the cost of the chain; required the cost of each.

16. Francis has 9 cents more than $\frac{1}{2}$ as many as Fannie, and they together have 42 cents; how many cents has each?

17. A cow and horse cost $132; required the cost of each, if the cow cost $\frac{2}{3}$ as much as the horse, minus 8 dollars.

18. A tree, whose length was 45 feet, was broken into two unequal parts, and $\frac{3}{5}$ of the longer piece, plus 5 feet, equals the shorter; required the length of each piece.

19. A man walked 110 miles in three days; he walked 5 miles further the second day than the first, and 10

miles farther the third day than the second; how far did he walk each day?

20. A man bought a sleigh, horse, and harness for $152; for the sleigh he gave twice as much as for the harness, plus $6, and for the horse 4 times as much as for the harness, plus $6; what did he pay for each?

21. In a certain field there are 42 animals, consisting of horses, sheep, and cows; required the number of each; provided $\frac{1}{2}$ of the number of sheep, + 10, equals the number of cows, and $\frac{1}{3}$ of the number of sheep, + 10, equals the number of horses.

LESSON VII.

A Number of Times one part Equals a Number of Times Another.

A AND B together have 34 apples, and $\frac{2}{3}$ of A's number equals $\frac{3}{4}$ of B's number; how many has each?

Solution.—If $\frac{2}{3}$ of A's number equals $\frac{3}{4}$ of B's, $\frac{1}{3}$ of A's equals $\frac{1}{2}$ of $\frac{3}{4}$, which is $\frac{3}{8}$ of B's; and $\frac{3}{3}$ of A's equals 3 times $\frac{3}{8}$, which are $\frac{9}{8}$ of B's; that is, $\frac{8}{8}$ of B's equals A's, which added to $\frac{9}{8}$ of B's, equals $\frac{17}{8}$ of B's, which equals 34 apples, etc.

2. The sum of two numbers is 28, and $\frac{1}{3}$ of the smaller equals $\frac{1}{4}$ of the greater; what are the numbers?

3. Thomas and Walton together have $55, and $\frac{2}{3}$ of Thomas's money equals $\frac{4}{5}$ of Walton's; how much has each?

4. Divide 46 oranges between Chester and Henry so that $\frac{3}{4}$ of Chester's may equal $\frac{2}{5}$ of Henry's number.

5. $\frac{3}{5}$ of the number of apple trees in an orchard equals $\frac{5}{7}$ of the number of peach trees, and in all there are 60 trees; required the number of each?

6. Twice the sum of two numbers is 30, and 3 times the smaller equals twice the greater; what are the numbers?

7. Two-thirds of the number of dollars that A and B

have equals 40; how many has each, if 5 times A's number equals 7 times B's number?

8. A pole, whose length was 63 feet, was broken into two parts, such that $\frac{3}{4}$ of the first part equals $\frac{3}{5}$ of the second; required the length of each piece.

9. Walter bought a hat and coat for \$26, and $2\frac{1}{2}$ times the cost of the hat equals $\frac{3}{4}$ of the cost of the coat; required the cost of each?

10. The difference between two numbers is 6, and $\frac{2}{5}$ of the first equals $\frac{4}{7}$ of the second; what are the numbers?

11. Fanny has 14 plums more than Sallie, and $\frac{2}{5}$ of Fanny's equals $\frac{3}{4}$ of Sallie's number; how many has each?

12. $\frac{1}{2}$ of the difference between two numbers is 6, and $\frac{1}{8}$ of the first number equals $\frac{1}{4}$ of the second; required the numbers.

13. Says B to C, $\frac{2}{3}$ of my age, + 6 years, equals $\frac{3}{4}$ of yours, and the sum of our ages is 42 years; required the age of each?

14. Five-sixths of the difference between A's and B's fortune is \$500, and $\frac{2}{5}$ of A's equals $\frac{4}{7}$ of B's fortune; what is the fortune of each?

15. A and B can build $\frac{1}{8}$ of a boat in a day, and twice what A builds equals what B builds; how much can each build in one day?

16. Two boys can do a piece of work in 6 days, and twice what A does equals what B does; how long will it take each to do it?

17. Three-fourths of 40 is $\frac{3}{5}$ of the number of apples and pears that Reuben has; how many has he of each, if 3 times the number of apples equals 7 times the number of pears?

18. Two pipes fill a cistern containing 144 gallons, and $\frac{2}{3}$ of what one pours in equals $\frac{2}{5}$ of what the other pours in; how much flows in through each pipe?

19. John and Henry can mow 30 acres of grass in a week, and $\frac{1}{2}$ of what John can mow in a day equals what Henry can mow in a day; how much does each mow in a week?

20. B can drink 6 quarts of mead in 4 days; $\frac{2}{3}$ of what B drinks equals $\frac{1}{2}$ of what A drinks, and also $\frac{1}{3}$ of what C drinks; in what time could A and C drink it alone?

LESSON VIII.

The Parts Proportioned to Given Numbers.

DIVIDE 30 cents between A and B, so that their shares will be to each other as 4 to 6.

Solution.—Since the shares are to be to each other as 4 to 6, if we divide 30 cents into $4 + 6$, which are 10 equal parts, 4 of these parts, or $\frac{4}{10}$ of 30, will be A's number, and 6 of these parts, or $\frac{6}{10}$ of 30, will be B's number, etc.

2. Divide 45 apples between Thomas and Harry, so that their shares will be to each other as 3 to 2.

3. Divide the number 50 into two parts, that shall be to each other as 7 to 3.

4. In a school, consisting of 45 pupils, there are 5 girls for every 4 boys; how many of each sex in the school?

5. The sum of two numbers is 40, and the larger is to the smaller as 5 to 3; required the numbers?

6. Divide 45 plums among three boys, so that their shares may be in the proportion 2, 3, and 4.

7. Two men bought a barrel of flour for $8, the first paying $3, and the second $5; how much of the flour belongs to each?

8. Three men bought 75 horses, and as often as the first paid $4 the second paid $5, and the third $6; how many horses should each receive?

9. Divide \$44 between A and B so that B shall have \$3½ as often as A has \$2.

10. The sum of two numbers is 50, and the first is to the second as ½ is to ⅓; what are the numbers?

11. Divide the number 49 into two parts which are to each other as ⅓ to ¼.

12. The sum of three numbers is 46; what is each of the numbers, if they are to each other as ½, ⅔, and ¾?

13. Divide the number 50 into 3 parts which shall be to each other as ⅔, 1½, and 2.

14. A, B, and C found a purse containing \$100, which they agree to divide in the proportion of ¾, ⅞, and 1½; how much does each receive?

15. A and B agree to pay \$250 toward building a church, which is to be situated 2 miles from A and 3 miles from B; how much does each contribute, if they pay in proportion to the reciprocals of their distances?

16. If \$420 be divided into two parts which are to each other as ½ to ⅔, it will respectively give ⅔ of A's and ⅘ of B's fortune; required the fortune of each.

17. If \$500 be divided into two parts to each other as 2 to 3, it will respectively give ⅔ of A's and ¾ of B's fortune; required the fortune of each.

18. A's fortune, added to ½ of B's fortune, equals \$2000; what is the fortune of each, provided A's fortune is to B's as 3 to 4?

19. One-third of A's fortune, plus ¼ of B's fortune, amounts to \$500; what is the fortune of each, if A's fortune is to B's as 9 to 8?

20. M's fortune, + ¾ of N's, which is equal to ½ of M's, is \$900; and if the sum of M's and N's be divided in the proportion of ½ to ⅔, it will respectively give ½ of R's and ⅔ of T's fortune; required the fortune of each.

LESSON IX.

Compound and Continued Proportion.

IF $\frac{2}{3}$ of a yard of cloth cost $\frac{4}{5}$ of a dollar, what will $\frac{3}{4}$ of a yard cost?

2. If 8 horses eat a quantity of hay in 16 weeks, how long will it last 32 horses?

3. If 5 men earn 30 dollars in a certain time, how much will 8 men earn in $\frac{1}{2}$ the time?

4. If 6 persons spend $36 in 3 days, how much will 10 persons spend in 5 days?

5. How long will 5 tons of hay last 8 horses, if 6 horses eat it in 12 weeks?

6. How long will 3 barrels of flour last 10 persons, if 4 persons eat 4 barrels in 40 weeks?

7. If 7 men can earn $28 in 4 days, how many dollars can 9 men earn in 6 days?

8. How long will 6 men require to build 6 boats, if 7 men build 3 boats in 12 weeks?

9. If 10 oxen eat 4 acres of grass in 6 days, in how many days will 30 oxen eat 8 acres?

10. If it required 4 men 7 days to perform a certain piece of work, how many men can perform a piece, 3 times as large, in 6 days?

11. If it require 5 men 8 days to build 20 rods of wall, how many men can, in 2 days, build $\frac{1}{2}$ as much wall?

12. How many men, in 10 days of 6 hours each, can earn as much as 6 men, in 20 days of 8 hours each?

13. How many oxen will eat 5 tons of hay in 5 weeks, if 12 oxen eat 4 tons in 4 weeks?

14. If 3 horses, in $\frac{1}{4}$ of a month, eat $\frac{3}{4}$ of a ton of hay, how long will $\frac{5}{6}$ of a ton last 5 horses?

15. If 4 men can do a piece of work in 6 days, in what time will it be completed if they receive the assistance of 5 men when the work is half done?

16. How many cents are 10 melons worth, if 4 melons are worth 8 oranges, and 3 oranges are worth 9 cents?

17. How many cents will 5 oranges cost, if 3 oranges are worth 9 apples, and 4 apples are worth 8 cents?

18. How many dollars will 10 sheep cost. if 5 sheep are worth 2 cows, and 4 cows are worth $80?

19. How many pigs can a man get for 2 cows, if 12 pigs are worth 3 sheep, and 12 sheep are worth 3 cows?

20. How many oranges can you buy for 20 cents, if 4 oranges are worth 8 apples, and 4 apples are worth 8 cents?

21. How many hens can you purchase for $12, if 4 hens are worth 2 turkeys, and 3 turkeys are worth $6?

22. If 6 sheep are worth 2 cows, and 10 cows are worth 5 horses; how many sheep can you buy for 3 horses?

23. If a measure of flour makes 5 four-cent loaves, how many 2-cent loaves will it make? How many 5-cent loaves will it make?

24. If a certain sum of money buys 9 four-cent oranges, how many 6-cent oranges can you buy for the same sum?

25. If a 5-cent loaf weighs 7 ounces when flour is worth 6 dollars a barrel, how much should it weigh when flour is worth 7 dollars per barrel?

26. If a 3-cent loaf weighs 9 ounces when flour is 6 dollars a barrel, how much ought a 4-cent loaf to weigh when flour is 8 dollars a barrel?

27. If 5 horses can eat a lot of grain in 12 days, in what time will it be consumed if 7 horses are added when the grain is $\frac{1}{3}$ eaten?

28. If 8 boys can weed a garden in 5 hours, in what time will the job be completed provided 3 boys leave when the work is half done?

29. If 9 men build 10 rods of wall in 8 days, in what time can 20 rods be built if $\frac{2}{3}$ of their number leave when the work is $\frac{1}{4}$ part completed?

SECTION VI.

PERCENTAGE.

Percentage is the process of computing by hundredths.

Per cent., from the Latin *per*, by, and *centum*, the hundred, means by or on the hundred.

Thus, 5 per cent. of a number of apples is 5 apples of a hundred; 10 per cent. of a number of dollars is 10 dollars of a hundred, and so on, whatever be the denomination.

In Percentage there are four quantities considered:

1. The *base*, or number on which percentage is estimated.
2. The *rate*, or number denoting how many of a hundred.
3. The *percentage*, denoting how many of the base.
4. The *amount* or *difference* of the base and percentage.

LESSON I.

To Find the Percentage.

AT a gain of 10 per cent., what part of the value equals the gain?

Solution.—A gain of 10 per cent. is a gain of 10 on 100. If on 100 the gain is 10, on 1 it is $\frac{1}{100}$ of 10, which is $\frac{10}{100}$, or $\frac{1}{10}$. Therefore, at a gain of 10 per cent., $\frac{1}{10}$ of the value equals the gain.

2. At a gain of 2, 4, 5, or 8 per cent., what part of the cost equals the gain?

3. At a loss of 12, 14, 16, or 20 per cent., what part of the value equals the loss?

4. If I gain 25, 30, or 35 per cent. on an investment, what part of the money invested equals the gain?

5. A gains 50 per cent. on his capital; what part of the capital equals the gain?

6. What part of the cost equals the gain at $8\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$, or $33\frac{1}{3}$ per cent.?

7. A man paid \$150 for a horse, and sold it at a gain of 10 per cent.; what was the gain?

Solution.—At a gain of 10 per cent., $\frac{10}{100}$, or $\frac{1}{10}$, of the cost equals the gain. $\frac{1}{10}$ of \$150 is \$15. Therefore, etc.

8. A lady bought a shawl for \$8, and sold it at a gain of 25 per cent.; required the gain.

9. Henry sold a cow worth \$40 at a loss of 5 per cent.; what did he receive for the cow?

10. A merchant sold 20 per cent. of 50 barrels of flour; how many barrels remained?

11. Samuel spent 20 per cent. of \$50 for a watch, and 20 per cent. of the remainder for a chain; how much had he remaining?

12. Which is the greater, and how much, 20 per cent. of 50 apples, or 6 times 4 per cent. of 25 apples?

13. Thomas, having a horse which cost \$120, sold it at a gain of 25 per cent., and the buyer sold it at a loss of 20 per cent.; what did the latter receive for it?

14. A lady bought 6 yards of calico for 180 cents, and sold it at a gain of 10 per cent.; what was the gain on each yard?

15. 10 per cent. of \$300 is $\frac{2}{5}$ of what Mary paid for a shawl; required the cost of the shawl.

16. A merchant purchased 10 barrels of flour for \$50, and sold them at a loss of 20 per cent.; what did he receive for each barrel?

17. 8 per cent. of \$200 is $\frac{2}{5}$ of what A gave for a watch; he sold it so as to gain 20 per cent.; for what did he sell it?

18. A and B together have \$1600, of which A owns $\frac{3}{5}$ as much as B; A then obtains 20 per cent. of B's part; how much does each now possess?

19. A owned 50 acres of land, and B owned three times as much; A sold B 20 per cent. of his land, and then bought 25 per cent. of B's; how much had each after this operation?

LESSON II.

To Find the Rate.

A MAN bought a watch for $20, and sold it for $25; what was the gain per cent.?

Solution.—If he bought it for $20, and sold it for $25, he gained the difference between $25 and $20, which is $5. If on $20 he gained $5, on $1 he gained $\frac{1}{20}$ of $5, which is $\frac{5}{20}$, or $\frac{1}{4}$, and on $100 he would gain 100 times $\frac{1}{4}$, which are $\frac{100}{4}$, or $25; hence the gain is 25 per cent.

2. A boy gave 25 cents for a knife, and sold it for 30 cents; what did he gain per cent.?

3. A lady bought a shawl for $5, and sold it for $8; what was the gain per cent.?

4. Thompson bought a boat for $20, and sold it for $16; what was the loss per cent.?

5. Rose bought a dress for $4, and sold it for $6; what was the gain per cent.?

6. Edwin bought a horse for $150, and sold it for $\frac{3}{5}$ of the cost; required the loss per cent.

7. Robert sold his horse for $150, which was $\frac{3}{4}$ of what he paid for it; what per cent. did he lose?

8. Elihu bought 10 cows for $200, and sold 8 of them for what they all cost; what was the gain per cent.?

9. What per cent. of $25 is $5? Of $40 is $8?

Solution.—$25 is 100 per cent. of itself, and $5, which is $\frac{1}{5}$ of $25, is $\frac{1}{5}$ of 100 per cent., or 20 per cent. of $25.

10. What per cent. of 16 is 4? Of 30 is 5? Of 200 is 8? Of 80 is 4? Of 96 is 12?

11. James, having 50 marbles, sold 20 per cent. of them; what per cent. of the whole remained?

12. A man bought 25 barrels of flour; he lost 20 per cent. of it, and sold 25 per cent of the remainder; what per cent. of the whole remained?

13. What per cent. of $\frac{1}{4}$ is $\frac{1}{8}$? Of $\frac{1}{2}$ is $\frac{1}{10}$? Of $\frac{1}{3}$ is $\frac{1}{8}$? Of $\frac{3}{4}$ is $\frac{3}{16}$? Of $\frac{4}{5}$ is $\frac{1}{10}$? Of $\frac{2}{3}$ is $\frac{2}{5}$?

14. If a miller takes $3\frac{1}{5}$ quarts of every bushel he grinds for toll, what per cent. does he take for toll?

15. $\frac{2}{3}$ of \$6 is twice what per cent. of $\frac{4}{5}$ of 50 dollars?

16. $\frac{4}{5}$ of \$10 is $\frac{1}{2}$ of what per cent. of $\frac{4}{5}$ of 50 dollars?

17. Sixty dimes is $\frac{6}{5}$ of what Samuel paid for 10 books; he sold them for 3 dimes apiece; required the loss per cent.

18. A merchant bought 30 barrels of flour for \$5 each, and sold $\frac{2}{3}$ of them at the rate of 3 barrels for \$24, and the rest for cost; required the gain per cent.

19. $\frac{1}{2}$ of 10 per cent. is what per cent. of 20 per cent.?

20. $\frac{3}{4}$ of 8 per cent. is what per cent. of 30 per cent.?

21. $\frac{2}{5}$ of 15 per cent. is what per cent. of 72 per cent.?

22. Mary sold some silk for \$12, and thereby cleared $\frac{1}{3}$ of this money; what would she have lost per cent. if she had sold it for 6 dollars?

23. A man sold a cow for \$25, and thereby cleared $\frac{1}{5}$ of this money; how much would he have gained per cent. if he had sold it for \$30?

24. Willis sold some books for \$12, and thereby cleared $\frac{1}{5}$ of the cost; what would he have lost per cent. by selling them for \$8?

LESSON III.

To Find the Base.

THOMAS sold his watch for \$25, and thereby gained 25 per cent.; what was the cost of the watch?

Solution.—If he gained 25 per cent., then $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which added to $\frac{4}{4}$, the cost, is $\frac{5}{4}$ of the cost, which equals \$25. If $\frac{5}{4}$ of the cost equals \$25, $\frac{1}{4}$ of the cost equals $\frac{1}{5}$ of \$25, which is \$5, and $\frac{4}{4}$, or the cost, equals 4 times \$5, which are \$20. Therefore, etc.

2. Mary sold her shawl for \$14, which was at a gain of 40 per cent.; required the cost of the shawl.

3. A farmer sold a cow for \$23, and thereby gained 15 per cent.; required the value of the cow.

4. A student sold his library for $140, and thereby lost 30 per cent.; what was its value?

5. By selling a hat for $8, Mary lost 20 per cent.; what was the value of the hat?

6. A dog was bought for $15, and sold at a gain of 20 per cent.; for what was it sold?

7. Mason gained 20 per cent. by selling cloth at $6 per yard; how should he have sold it to gain 25 per cent.?

8. If by selling land at $75 an acre I gain 25 per cent., how must I sell it to lose 40 per cent.?

9. If a merchant sells muslin at 39 cents a yard, and thereby gains 30 per cent., for what shall he sell it to lose 40 per cent.?

10. A boat was sold for $91, which was at a loss of 35 per cent.; how should it have been sold to gain 40 per cent?

11. Taylor lost 60 per cent. on a watch by selling it for $40; what should he have received for it to gain 60 per cent.?

12. Hinkston sold his horse and carriage for $240, and thereby lost 4 per cent.; what would he have gained per cent. by selling it for $300?

13. A wagon was sold for $90, which was 10 per cent. less than its value; what would have been the gain per cent. if it had been sold for $120?

14. Mr. Bowman sold 2 books for $15 each; on one he gained 25 per cent., and on the other he lost 25 per cent.; how much did he lose by the transaction?

15. A tailor sold 2 coats for $12 each; on one he gained 20 per cent., and on the other he lost 20 per cent.; did he gain or lose by the sale, and how much?

16. B bought a watch for $42, which was 40 per cent. less than its value; he sold it for 30 per cent. more than its value; what was the gain?

17. A man sold 2 watches for $80 each; on one he lost 20 per cent., and on the other he gained 25 per cent.; how much was gained or lost by the transaction?

18. A merchant sold a stove for $30, and thereby lost 25 per cent.; he then bought another for $30, and upon it gained 25 per cent.; what was the gain or loss?

19. Martha sold a painting so that $\frac{2}{3}$ of what she received for it equaled $\frac{4}{5}$ of the cost; did she gain or lose, and how much per cent.?

20. Terrel sold his watch and chain for $120, receiving 5 times as much for the watch as for the chain; on the watch he gained 25 per cent., and on the chain he lost 20 per cent.; what was the gain?

LESSON IV.

To Find the Base.

A MAN gained 25 per cent. by selling his watch for $20 more than it cost; required the cost.

Solution.—At a gain of 25 per cent., $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which is $20; if $\frac{1}{4}$ of the cost equals $20, $\frac{4}{4}$ of the cost equals 4 times $20, or $80.

2. A farmer gained 30 per cent. by selling a cow for $9 more than she cost; what did the cow cost?

3. A, by selling his dog for $6 less than it cost, lost 15 per cent; required the cost of the dog.

4. A hat was sold for 20 cents less than cost, which was at a loss of 40 per cent.; required its cost.

5. Four is 10 per cent., 5 is 20 per cent., and 6 is 25 per cent., of what numbers?

6. Eight is 40 per cent., 9 is 30 per cent., and 12 is 12 per cent., of what numbers?

7. Thirty is 25 per cent. less, and 25 per cent. more, than what numbers?

8. A man gained $20 by selling a boat for 20 per cent. more than its value; what would he have gained by selling it for 10 per cent. above its value?

9. A piano was sold for \$60 less than its value, which was at a loss of 30 per cent.; what would have been the gain per cent. if it had been sold for \$250?

10. \$24 is 4 per cent. of the sum of A's and B's fortune; how much money has each, provided A has twice as much as B?

11. An agent receives \$120 to purchase goods, after deducting his commission, which is 20 per cent. on the amount expended; required his commission.

REMARK.—$\frac{1}{5}$, his commission, + $\frac{5}{5}$, what he expended, = $\frac{6}{5}$ of what he expended, which is \$120.

12. A man receives 25 per cent. for purchasing goods; how many dollars' worth can he purchase with \$200, retaining his commission?

13. A receives \$216 to buy goods, and is to retain 8 per cent. on the money expended; required the amount of money expended.

14. Frick received \$2800 to invest in land, after deducting his commission, which is 12 per cent. on the amount invested; required his commission.

15. How much grain must a farmer take to mill that he may bring away the flour of 3 bushels, after the miller has taken 10 per cent. of all he took there?

16. A's shop is valued at \$900; for what sum must he have it insured, at 10 per cent., so that in case of loss he may receive both the value of the shop and premium?

17. At 2 per cent., what must a house worth \$4900 be insured for, so that the premium may be included in case of loss?

18. How many yards of cloth, at \$4 a yard, must a merchant buy, that by selling it at a profit of 20 per cent. he may gain \$8?

19. A man receives \$530 to purchase sheep and cows; what sum will he expend for each, after deducting his commission, which is 6 per cent. of the money expended, provided he expends 4 times as much for cows as sheep?

LESSON V.

Interest.

1. *Interest* is money charged for the use of money or property. It is estimated at a certain rate per cent. per annum.

2. The *principal* is the sum on which interest is computed.

3. The *amount* is the sum of the principal and interest.

4. The *rate per cent.* is the interest of 100 for one year.

5. In computing interest we shall consider 30 days to the month, and 12 months to the year.

REDUCE 2 years and 6 months to the fraction of a year.

Solution.—In 1 year there are 12 months; hence, 1 month is $\frac{1}{12}$ of a year, and 6 months are 6 times $\frac{1}{12}$, which are $\frac{6}{12}$, or $\frac{1}{2}$ of a year, which, added to 2 years, equals $2\frac{1}{2}$, or $\frac{5}{2}$ years.

2. Reduce each of the following to a fraction of a year: 2 yr., 8 mo.; 3 yr., 4 mo.; 4 yr., 3 mo.

3. How many years in 3 yr., 9 mo.? 7 yr., 2 mo.? 4 yr., 10 mo.? 6 yr., 5 mo.? 6 yr., 8 mo.?

4. Reduce 3 years, 7 months, 15 days, to the fraction of a year.

Solution.—There are 30 days in a month, hence 1 day is $\frac{1}{30}$, and 15 days are $\frac{15}{30}$, or $\frac{1}{2}$, of a month, which, added to 7 months, equals $7\frac{1}{2}$, or $\frac{15}{2}$ months, etc.

5. How many years in 2 yr., 2 mo., 2 da.? 3 yr., 3 mo., 9 da.? 2 yr., 4 mo., 5 da.?

6. How many years in 4 yr., 7 mo., 6 da.? 5 yr., 5 mo., 10 da.? 6 yr., 2 mo., 12 da.?

7. How many years in 7 yr., 3 mo., 18 da.? 8 yr., 6 mo., 20 da.? 2 yr., 1 mo., 6 da.?

8. At 5 per cent. for 4 years, what part of the principal equals the interest?

Solution.—At 5 per cent., $\frac{5}{100}$ of the principal equals the interest for 1 year, and for 4 years, 4 times $\frac{5}{100}$, which are $\frac{5}{25}$, or $\frac{1}{5}$.

9. At 10 per cent. for 5 years, what part of the principal equals the interest?

10. At 8 per cent. for 5 years, what part of the principal equals the interest?

11. At 6 per cent. for 2 yr. and 4 mo., what part of the principal equals the interest?

12. At 6 per cent. for 5 yr. and 8 mo., what part of the principal equals the interest?

13. At 8 per cent. for 1 yr., 4 mo., 15 da., what part of the principal equals the interest?

14. What is the interest of $60 for 6 yr., at 5 per cent.?

15. What is the interest of $40 for 4 yr., at 5 per cent.?

16. What is the interest of $30 for 5 yr., at 4 per cent.?

17. What is the interest of $75 for 8 yr., at 6 per cent.?

18. What is the interest of $50 for 9 yr., at 8 per cent.?

19. What is the interest of $28 for 10 yr., at 5 per cent.?

20. What is the interest of $400 for 11 yr., at 5 per cent.?

21. What is the interest of $200 for $6\frac{1}{2}$ yr., at 6 per cent.?

22. What is the interest of $300 for $5\frac{2}{3}$ yr., at 9 per cent.?

What is the interest

23. Of $600 for 2 years, 3 months, at 8 per cent.?

24. Of $300 for 4 years, 6 months, at 6 per cent.?

25. Of $240 for 3 years, 9 months, at 8 per cent.?

26. Of $225 for 6 years, 8 months, at 6 per cent.?

27. Of $500 for 5 years, 4 months, at 9 per cent.?

28. Of $330 for 7 years, 6 months, at 4 per cent.?

29. Of $222 for 8 years, 4 months, at 6 per cent.?

30. Of $666 for 6 years, 3 months, at 8 per cent.?

31. Of $288 for 4 years, 2 months, at 12 per cent.?

32. Of $440 for 2 years, 1 month, at 12 per cent.?

33. Of $120 for 5 years, 10 months, at 12 per cent.?

34. Of $500 for 3 yr., 7 mo., 6 da., at 5 per cent.?

35. Of $300 for 5 yr., 3 mo., 18 da., at 10 per cent.?

36. Of $400 for 2 yr., 3 mo., 9 da., at 40 per cent?

37. Of $500 for 2 yr., 2 mo., 12 da., at 5 per cent.?

38. Of $600 for 1 yr., 6 mo., 12 da., at 15 per cent.?

39. Of $200 for 1 yr., 6 mo., 20 da., at 9 per cent.?

LESSON VI.

To Find the Amount.

WHAT is the amount of $50 for 5 years, at 8 per cent. ?

Solution.—For 5 years at 8 per cent., $\frac{40}{100}$, or $\frac{2}{5}$, of the principal equals the interest, which, added to $\frac{5}{5}$ of the principal, equals $\frac{7}{5}$ of the principal, or the amount. $\frac{1}{5}$ of $50 is $10, etc.

2. What is the amount of $250 for 4 years, at 5 per cent. ?

3. What is the amount of $120 for 7 years, at 10 per cent. ?

4. What is the amount of $400 for 5 years, at 7 per cent. ?

5. What is the amount of $200 for 2 years, 3 months, at 8 per cent. ?

6. What is the amount of $600 for 7 years, 6 months, at 6 per cent. ?

7. What is the amount of $300 for 3 years, 9 months, at 8 per cent. ?

8. What is the amount of $300 for 8 years, 10 months, at 6 per cent. ?

9. What is the amount of $360 for 8 years, 4 months, at 9 per cent. ?

10. What is the amount of $100 for 2 years, 6 months, 20 days, at 9 per cent. ?

11. A and B wish to divide the amount of $500 for 8 years, at 5 per cent., so that A's part shall be 6 times B's; required the share of each.

12. The amount of $250 for 6 years, at 10 per cent., is to be divided between C and D, so that C shall have 3 times as much as D; what does each receive?

13. James and Henry have $1500 on interest for 4 years, at 10 per cent.; what amount of interest will each receive, provided James has twice as much as Henry?

14. A's fortune is \$200, which is $\frac{1}{4}$ of B's; what interest will each receive on his money in 4 years, at 5 per cent.?

15. C's money is \$300, which is $\frac{3}{4}$ of D's money; what is the amount of the money of each on interest, for 5 years, at 6 per cent.?

16. A's money is \$400, which is $\frac{2}{3}$ of B's money; how much more interest will B receive than A in 8 years, at 5 per cent.?

17. A, B, and C, together, have \$1200, of which A has twice, and B 3 times, as much as C; what is the interest of each for 5 years, at 6 per cent.?

18. If the interest of \$2500 for 4 years, at 10 per cent., be divided into two parts, which are as 2 to 3, it will respectively give $\frac{1}{8}$ of B's and $\frac{1}{2}$ of A's money; how much has each?

LESSON VII.

To Find the Principal.

WHAT principal will, in 6 years, at 5 per cent., give \$60 interest?

REMARK.—We find $\frac{3}{10}$ of the principal equals the interest, which is \$60. If $\frac{3}{10}$ of the principal equals \$60, $\frac{1}{10}$ equals $\frac{1}{3}$ of \$60, which is \$20, and $\frac{10}{10}$, the principal, equals 10 times \$20, which are \$200.

2. What principal will, in 7 years, at 5 per cent., give \$21 interest?

3. What principal will, in 8 years, at 6 per cent., give \$12 interest?

4. What principal will, in 3 years, at 8 per cent., give \$60 interest?

5. What principal will, in 7 years, at 4 per cent., give \$70 interest?

6. What principal will, in 8 years, at 5 per cent., give \$60 interest?

7. What principal will, in 3 years and 4 months, at 6 per cent., give \$80 interest?

8. A man pays $360 interest, at 6 per cent., annually on money borrowed; what is the sum borrowed?

9. How much money must a person borrow, that he must pay an annual interest of $150, at 5 per cent.?

10. The interest of $\frac{2}{3}$ of A's money for 6 years and 3 months, at 4 per cent., is $250; what is his money?

11. How much money has Howard on interest, supposing he receives $320 for 5 years, 4 months, at 6 per cent.?

12. The interest of $\frac{3}{5}$ of A's and $\frac{3}{4}$ of B's fortune, for 5 years, at 6 per cent., is $60 and $90, respectively; required the fortune of each.

13. The interest of the sum of A's and B's fortune, for 5 years, at 7 per cent., is $210; what is the fortune of each, provided B is worth twice as much as A?

14. Howard's money is 3 times Howell's, and in 5 years, at 8 per cent., Howard receives $600 interest; how much money has each?

15. The interest on $\frac{1}{2}$ of A's and $\frac{1}{3}$ of B's fortune, for 5 years, at 6 per cent., is $240; what is the fortune of each, provided $\frac{1}{2}$ of A's equals $\frac{1}{3}$ of B's?

16. A's money is 4 times B's, and the sum of the interest received by both for 3 years, at 8 per cent., is $600; how much money has each?

17. The interest for 4 years, at 5 per cent., on the money Martin owes, is $40; and the interest for the same time and rate per cent., on the money due him, is $70; how much more has he due than he owes?

18. The interest on the money A paid for a farm, house, and store, for 8 years, at 5 per cent., equals $18,000; what was the cost of each, provided the farm cost 3 times as much as the house, and the house twice as much as the store?

LESSON VIII.

To Find the Principal.

1. THE *present worth* of a debt payable at some future time, without interest, is such a sum as would, at a given rate per cent., *amount* to the debt at the time it becomes due.

2. The *discount* is the allowance made for the payment of money before it is due. The *present worth* may be found in the same manner as the principal, when we have given the amount, time, and rate per cent. The discount equals the debt minus the present worth.

WHAT principal will, in 8 years, at 5 per cent., amount to $140?

Solution.—At 5 per cent., for 8 years, $\frac{40}{100}$, or $\frac{2}{5}$, of the principal equals the interest, which, added to $\frac{5}{5}$, the principal, equals $\frac{7}{5}$ of the principal, which equals the amount, or $140, etc.

2. What principal will, in 7 years, at 6 per cent., amount to $710?

3. What principal will, in 4 years, at 10 per cent., amount to $420?

4. What principal will, in 6 years and 8 months, at 9 per cent., amount to $320?

5. What principal will, in 5 years and 10 months, at 6 per cent., amount to $540?

6. What is the present worth of $60, due 4 years hence, at 5 per cent.?

7. What is the present worth of $52, due 5 years hence, at 6 per cent.?

8. The amount of $\frac{2}{3}$ of B's fortune, for 3 years and 4 months, at 6 per cent., is $600; what is his fortune?

9. What is the present worth of $270, due 7 years hence, at 5 per cent.?

10. What is the present worth of $370, due 8 years hence, at 6 per cent?

11. What is the discount of $580, due 9 years hence, at 5 per cent.?

12. What are the present worth and discount of $340, due 10 years hence, at 7 per cent.?

13. The amount due on a note which had been on interest for 3 years and 4 months, at 9 per cent., is $520; required the face of the note.

14. The sum of A's and B's money, being on interest for 3 years and 9 months, at 8 per cent., amounts to $2600; what is the money of each, if A's is 3 times B's?

15. A's money, added to B's, being on interest for 5 years and 4 months, at 6 per cent., amounts to $660; what sum has each, if A's is 4 times B's?

16. A man wishes to place such a sum of money on interest, at 6 per cent., that it will give an annual interest of $360 for a poor sister; required the amount invested.

17. Four times A's money, added to 3 times B's, being on interest for 4 years, at 10 per cent., amounts to $4200; how much has each, if 3 times B's equals A's?

18. Two thirds of A's fortune, plus $\frac{3}{4}$ of B's, being on interest for 6 years, at 5 per cent., amounts to $7800; what is the fortune of each, supposing $\frac{2}{3}$ of A's equals $\frac{3}{4}$ of B's?

19. $\frac{1}{2}$ of the cost of Bowman's house, plus $\frac{2}{3}$ of the cost of his farm, being on interest for 5 years, at 8 per cent., amounts to $2100; what is the cost of each, provided the house cost $\frac{1}{3}$ as much as the farm?

20. Two times the value of a horse, plus 3 times the value of a cow, which is $\frac{1}{2}$ of the value of the horse, in 8 years, at 5 per cent., gives $84 interest; required the value of each.

21. The money Henry paid for a horse, carriage, and harness, in 10 years, at 5 per cent., would give such an interest that if put on interest for the same time and rate it would amount to $270; how much did he pay for each, if the horse cost twice as much as the carriage, and the carriage 3 times as much as the harness?

LESSON IX.

To Find the Time.

THE interest of $200, for a certain time, at 5 per cent., is $60; required the time.

Solution.—At 5 per cent. for *one* year, $\frac{5}{100}$, or $\frac{1}{20}$, of the principal equals the interest. $\frac{1}{20}$ of $200 is $10. If it require one year for $200 to gain $10, to gain $1 it will require $\frac{1}{10}$ of a year, and to gain $60 it will require 60 times $\frac{1}{10}$ of a year, which are $\frac{60}{10}$, or 6 years.*

2. In what time will $100, at 6 per cent., give $21 interest?

3. In what time will $100, at 7 per cent., give $14 interest?

4. In what time will $200, at 5 per cent., give $40 interest?

5. In what time will $150, at 6 per cent., give $45 interest?

6. In what time will $100, at 8 per cent., give $32 interest?

7. In what time will $300, at 10 per cent., give $120 interest?

8. In what time will $200, at 8 per cent., give $48 interest?

9. In what time will $60, at 5 per cent., give $21 interest?

10. In what time will $25, at 6 per cent., give $9 interest?

11. In what time will $50, at 9 per cent., give $30 interest?

12. In what time will $150, at 5 per cent., amount to $210?

13. In what time will $300, at 7 per cent., amount to $510?

* The latter part of this may be given thus: It will require as many years as $10 is contained times in $60, which are 6.

14. In what time will a principal gain twice itself, at 40 per cent.?

REMARK.—We find a principal gains $\frac{2}{5}$ of itself in 1 year, and to gain twice itself will require as many years as $\frac{2}{5}$ is contained times in 2, or 5 years.

15. In what time will a principal gain 3, 4, and 5 times itself, at 10 per cent.?

16. In what time will a principal double itself, at 5 per cent.? At 6? 7? 8? 9? 10? $12\frac{1}{2}$? 15?

17. In what time will a principal treble itself, at 5 per cent.? At 10? 20? 25? 40? 50?

18. In what time will a principal quadruple itself, at 5 per cent.? At 15? 30? 50? 60? 100?

19. The amount of a certain principal, for a certain time, at 5 per cent., is \$250, and the amount for the same time, at 8 per cent., is \$280; required the principal and time.

20. A certain sum of money, on interest, amounts, in a certain time, at 6 per cent., to \$310, and at 10 per cent., for the same time, to \$350; required the time and principal.

LESSON X.

To Find the Rate.

AT what per cent. will \$60, in 5 years, give \$21 interest?

Solution.—For 5 years, at *one* per cent., $\frac{5}{100}$, or $\frac{1}{20}$, of the principal equals the interest. $\frac{1}{20}$ of \$60 is \$3. If \$60 in 5 years, at 1 per cent., gains \$3, to gain \$21 it will require as many times 1 per cent. as \$3 is contained times in \$21, or 7 per cent.

2. At what per cent. will \$40, in 5 years, give \$20 interest?

3. At what per cent. will \$200, in 3 years, give \$36 interest?

4. At what per cent. will \$300, in 4 years, give \$60 interest?

5. At what per cent. will \$80, in 5 years, give \$32 interest?

6. At what per cent. will \$50, in 6 years, give \$15 interest?

7. At what per cent. will \$60, in 7 years, give \$21 interest?

8. At what per cent. will \$100, in 4 years, amount to \$120?

9. At what per cent. will \$90, in 5 years, amount to \$117?

10. At what per cent. will \$6, in 3 yr. 4 mo., amount to \$7?

11. At what per cent. will a given principal gain 3 times itself in 10 years?

REMARK.—A principal gains $\frac{1}{10}$ of itself at 1 per cent., and to gain 3 times itself it will require as many times 1 per cent. as $\frac{1}{10}$ is contained times in 3, which are 30.

12. At what per cent. will a principal gain 2, 4, 5, and 6 times itself in 30 years?

13. At what per cent. will a principal double itself in 4 years? In 10? 12? 20? 25? $33\frac{1}{3}$? 50?

14. At what per cent. will a principal treble itself in 10 years? In 20? 25? 40? 80? 100?

15. At what per cent. will a principal quadruple itself in 10 years? In 15? 30? 60? 100? 150?

16. At what per cent. will a principal quintuple itself in 4 years? In 20? 40? 80? 100? 200?

17. The amount of a certain principal for 7 years, at a certain per cent., is \$540, and for 10 years, \$600; what are the principal and rate per cent.?

18. The amount of a certain principal for 4 years, at a certain per cent., is \$420, and for 9 years, at the same rate, \$570; required the rate per cent. and principal.

SUGGESTION.—Review Sections V. and VI., unless they are thoroughly understood.

SECTION VII.

THIS Section is designed to complete the course in Mental Arithmetic for graded schools, where the time for study is limited.

Teachers who prefer may have a still shorter course by taking the problems in each lesson up to the one marked with a *.

LESSON I.

Pasture Problems.

A AND B hired a pasture for $36; A pastured 4 cows and B 5 cows; how much should each pay?

Solution.—If A pastured 4 cows and B 5, they both pastured 4 + 5, which are 9 cows. If the pasturage of 9 cows cost $36, the pasturage of 1 cow will cost $\frac{1}{9}$ of $36, which is $4, and the pasturage of 4 cows, A's number, will cost 4 times $4, or $16, and the pasturage of 5 cows, B's number, will cost 5 times $4, or $20.

2. Two boys bought 60 apples for 12 cents; one paid 5 cents and the other 7 cents; how many apples should each receive?

3. Rufus and William paid 20 cents for 40 peaches, of which Rufus paid 9 and William 11 cents; how many peaches belong to each?

4. Three men hired a horse for 20 days, at the rate of $1 per day; the first used it 5, the second 6, and the third 9 days; how much should each pay?

5. A and B hired a pasture for $44; A puts in 12 oxen, and B 100 sheep; how much should each pay, supposing an ox to eat as much as 10 sheep?

6. Two farmers hired a pasture for $56; one turns in 10 cows, and the other 36 horses; how much should each pay, provided a cow eats twice as much as a horse?

7. Three men, A, B, and C, bought 144 bushels of peaches for $72, of which A paid $\frac{1}{8}$, B $\frac{2}{3}$, and C the remainder; how many bushels did each receive?

8. A and B engage to do a piece of work for $72; A sends 6 men, and B 15 boys; how much should each receive, supposing 2 men to do as much as 3 boys?

9. A and B agree to mow a field of grass for $54; A sends 3 men 5 days, and B sends 4 men 3 days; how much should A and B receive respectively?

10. Two men hire a lot of pasture for $10; one turns in 6 horses for 7 days, and the other 7 horses for 4 days; how much should each pay?

11. A and B built a boat for $140; A sent 6 men 5 days, and B 4 men 10 days; how much should A and B receive respectively?

12. Two men gain in trade $440; A put in $25 for 4 months, and B $15 for 8 months; what is each man's share of the gain?*

13. C and D build a wall for $120; C with 4 assistants labored 4 days, and D with 3 assistants labored 5 days; how much do C and D receive respectively?

14. A, B, and C build a boat for $62; A sent 3 men 4 days, B 4 men 5 days, C 5 men 6 days; how much do A, B, and C receive respectively?

15. E and F engaged to reap a field of wheat for $54; E sent 3 men 5 days, and F 6 boys 4 days; how much should each receive, if 1 man does as much as 2 boys?

16. In a field of grass, which cost $24, M turned 16 horses for 3 weeks, and N 25 cows for 4 weeks; how much should each pay, if 4 horses eat as much as 5 cows?

17. A and B plowed a field for $76; A employed 12 horses, and B 18 oxen; they completed it in 4 days; what was the value of the daily labor of each horse and ox, supposing 3 horses to do as much work as 5 oxen?

18. R, S, and T hire a pasture for $63; R puts in 6 cows, S puts in 18 horses, and T 48 sheep; how much should each pay, if a cow eats as much as 2 horses, and a horse as much as 4 sheep?

LESSON II.

Beggar and Equal Number Problems.

A GENTLEMAN gave 4 cents each to some little beggars; had he given them 7 cents each, it would have taken 36 cents more; how many beggars were there?

Solution.—By the second condition of the question he gave each beggar 7 — 4, which is 3 cents more than by the first, and to them all 36 cents more; hence there were as many beggars as 3 is contained times in 36, which are 12. Therefore, etc.

2. A teacher gave his pupils 2 questions each, and had 26 questions remaining; if he had given them each 4 questions, there would have been none remaining; required the number of pupils and questions.

3. A father gave his sons $5 each, and had $30 remaining; had he given them $8 each, it would have taken all his money; required the number of sons and amount of money.

4. Mary gave some beggars 6 cents each, and had 25 cents remaining; had she given them 8 cents each, she would have had 3 cents remaining; how many beggars were there?

5. Edward bought a certain number of melons, at the rate of 5 cents each; if he had paid 3 cents each, they would have cost 14 cents less; how many melons did he buy?

6. A lady, wishing to buy some ribbon, found if she bought that at 10 cents a yard, she would want 9 cents to pay for it, but if she bought that at 7 cents a yard, she would have 9 cents remaining; how much money had she?

7. A gentleman divided 28 apples between an equal number of boys and girls, giving to each girl 3 and to each boy 4 apples; required the number of boys and girls.

REMARK.—He gave to one girl and one boy 3 plus 4, or 7 apples; hence there were as many of each as 7 is contained times in 28, which are 4.

8. A man bought an equal number of pigs and sheep for $81, giving $4 each for the pigs, and $5 each for the sheep; how many of each did he buy?

9. A boy expended 36 cents for an equal number of melons and lemons, giving 4 cents each for the melons, and 2 cents each for the lemons; how many of each did he purchase?

10. Morris and Robert have each a certain number of peaches; if Morris had 10 more, he would have twice as many as Robert, but if he had 30 more, he would have 4 times as many as Robert; how many has each?

11. A drover bought a number of sheep at $3\frac{1}{2}$ a head, and found he lacked $6 of having money enough to pay for them; if he had paid $2 a head, he would have had $9 remaining; how much money had he?*

12. Sallie wishes to buy a silk dress; if she pays $3 a yard, she will lack $6 to pay for it, but if she pays $2.50 a yard, she will have $5 remaining; required her money, and the number of yards in the dress.

13. A lady gave 60 cents to some poor children; to each boy she gave 2 cents, and to each girl 4 cents; how many were there of each, provided there were 3 times as many boys as girls?

14. A, B, and C dig a ditch for $60; A receives $1\frac{1}{2}$, B $2, and C $2\frac{1}{2}$ a day; how many days were they employed, and what did each receive?

15. A and B agree to perform a piece of work, A receiving $2 and B $3 a day; A works twice as many days as B, and they together receive $70; how many days did each labor?

16. Two boys had an equal sum of money; one bought a certain number of oranges, at 4 cents each, and had 12 cents remaining; the other bought twice as many apples, for 3 cents each, and had 2 cents remaining; how much money had each?

LESSON III.

Working Problems.

A CAN do a piece of work in 4 days; what part of it can he do in one day?

2. B can cut a cord of wood in $\frac{1}{3}$ of a day; how much can he cut in one day?

3. A man can build $\frac{2}{3}$ of a boat in a week; how long will it require for him to build the whole boat?

4. A mason can build a wall in $2\frac{1}{3}$ days; what part of it can he build in one day?

5. If A and B can mow $\frac{4}{5}$ of a field of grass in one day, how long will it require to mow the whole field?

6. A can do a piece of work in 3 days, and B in 6 days; what part can each do in one day?

7. If B can do a piece of work in 3 days, and C in 6 days, how much can they together do in one day?

8. If B and C can do $\frac{2}{3}$ of a piece of work in one day, how long will it require to do the whole work?

9. Fuller can eat a peck of apples in 8 days, and Broadhead in 6 days; how many days would it last them both?

10. A can dig a ditch in 5 days, and B in 6 days; in what time will they do it working together?

11. C can make a chest in 4 days, and D in 7 days; in what time can they make it working together?

12. E can reap a field in 6 days, and F in 8 days; how long will it take them both to reap it?

13. A can do a piece of work in 3 days, B in 4 days, and C in 6 days; in what time can they together do it?

14. A cistern has two pipes, by the first of which it may be filled in 12 hours, and by the second in $\frac{2}{3}$ of the time; how long will both be in filling it?

15. A can make a bookcase in 6 days, and A and B can make it in 4 days; in what time can B make it alone?

16. A, B, and C can dig a ditch in 3 days. A can dig

it in 6 days, and B in 8 days; in what time can C alone dig it?

17. A pound of tea lasted a man and wife 3 months, and the wife alone 4 months; how long would it last the man alone?

18. A, B, and C can mow a field in 4 days, A and B in 6 days, and B and C in 9 days; how long will it take each to mow it? *

19. A, B, and C can dig a ditch in 6 days, A and B in 8 days, and B in 12 days; how long will it take each to dig it?

20. If 3 men, or 4 boys, can do a piece of work in 12 days, in what time can 3 men and 4 boys do it?

21. If A can do a piece of work in $\frac{1}{3}$ of a day, and B in $\frac{1}{4}$ of a day, how long will it take both to do it?

22. C can cut a cord of wood in $\frac{3}{4}$ of a day, and D in $\frac{4}{5}$ of a day; in what time can they together cut a cord?

23. D can make a fence in 9 days, and D and E in 6 days; how long will it take E to make what remains after D has built $\frac{2}{3}$ of it?

24. Two men, or 3 boys, can plow an acre in $\frac{1}{6}$ of a day; how long will it require 3 men and 2 boys to plow it?

25. Amos can plow 25 per cent. of a field in a month, and Anson 45 per cent.; after they both had worked 2 weeks, how long would it require Amos to finish it?

26. A, B, and C can mow a field in 6 days, and A and B in 9 days; after the three had worked 2 days, C left; how long did it require A and B to finish it?

27. Marie can make a dress in 6 days, Sallie in $\frac{1}{2}$ of the time, and Ewretta in $\frac{2}{3}$ of the time; in what time can Marie and Sallie finish it, after the three had worked $\frac{2}{3}$ of a day?

28. A, B, and C can build a vessel in $\frac{1}{4}$ of a year, A and C in $\frac{1}{2}$ of a year, and C in $\frac{2}{3}$ of a year; after they had all labored 1 month, A left; in what time could B and C finish it?

LESSON IV.

Labor and Fish Problems.

A MAN receives $3 a day for his labor, and forfeits $1 each day he is idle, and at the expiration of 30 days receives $50; how many days was he idle?

Solution.—Had he labored 30 days, he would have received 30 times $3, or $90; he therefore lost $90 — $50, which is $40, by his idleness. Every day he was idle he lost $3, his wages, plus $1, his forfeit, which are $4. If in 1 day he loses $4, to lose $40 it will require as many days as 4 is contained times in 40, which are 10. Hence he was idle 10 days.

2. A laborer agreed to work for $2 a day, on condition that every day he was idle he should forfeit 50 cents; how many days did he labor, if at the end of 25 days he received $30?

3. A boy agreed to carry 32 glasses to a certain store for 5 cents apiece, on condition that for each one he broke he should forfeit 10 cents; he received $1. How many did he break?

4. Robert agreed to carry 50 oranges to market for $\frac{1}{2}$ a cent each, on condition that he should forfeit $2\frac{1}{2}$ cents for each one he ate; he received 16 cents. How many did he eat?

5. James engaged to labor on condition that for every day he worked he should receive $1\frac{1}{2}$, and for every day he played he should pay $\frac{1}{2}$ for his board; at the expiration of 30 days he received $35. How many days did he work?

6. The head of a fish is 10 inches long, the tail is as long as the head, plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both; required the length of the fish.

Solution.—One-half of the length of the body, + 10 inches, equals the length of the tail, which, added to the length of the

head, equals $\frac{1}{2}$ of the length of the body, + 20 inches, which, by the condition of the problem, equals $\frac{2}{3}$ of the length of the body. If $\frac{2}{3}$ of the length of the body equals $\frac{1}{2}$ of the length of the body, + 20 inches, $\frac{2}{3} - \frac{1}{2}$, or $\frac{1}{6}$ of the length of the body, must equal 20 inches, etc.

7. The head of a fish is 8 inches long, the tail is as long as the head, plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both; required the length of the fish.

8. The head of a perch is 4 inches long, the tail is as long as the head, plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both; what is the length of the perch?

9. The tail of a pike weighs 3 ounces, the head weighs as much as the tail, plus $\frac{1}{4}$ of the weight of the body, and the body weighs twice as much as the head and tail; required the weight of the fish.*

10. The head of a trout weighs 2 pounds, the tail weighs 2 pounds more than the head, plus $\frac{1}{3}$ of the body, and the body weighs as much as the head and tail together; required the weight of the fish.

11. Warner receives $2.50 a day for his labor, and pays 50 cents a day for his board; at the expiration of 40 days he has saved $50; how many days did he work, and how many days was he idle?

12. A gentleman receives $4 a day for his labor, and pays $8 a week for his board; at the expiration of 10 weeks he has saved $144; required the number of idle and working days.

13. A agreed to labor a certain time for $60, on condition that for each day he was idle he should forfeit $2; at the expiration of the time he received $30; how many days did he labor, supposing he received $2 a day for his labor?

LESSON V.

Combination Problems.

IF 40 lb. of sea-water contain 2 lb. of salt, how much fresh water must be added to these 40 lb. so that 6 lb. of the new mixture may contain $\frac{1}{5}$ lb. of salt?

Solution.—If 6 lb. of the mixture contain $\frac{1}{5}$ lb. of salt, to contain 1 lb. of salt, there must be 5 times 6, or 30 lb. of the mixture, and to contain 2 lb. of salt there must be 2 times 30 lb., or 60 lb. of the mixture; hence there must be added 60 lb., minus 40 lb., or 20 lb. of water.

2. If 50 lb. of sea-water contain 2 lb. of salt, how much fresh water must be added to these 50 lb. so that 10 lb. of the new mixture may contain $\frac{1}{5}$ of a pound of salt?

3. In a mixture of silver and copper, consisting of 60 oz., there are 4 oz. of copper; how much silver must there be added that there may be $\frac{1}{3}$ oz. of copper in 6 oz. of the mixture?

4. In a drove of 100 animals, consisting of horses and cows, there are 40 cows; how many horses must I sell that there may be 5 horses to 4 cows?

5. In a school of 80 pupils there are 32 girls; how many boys must leave that there may be 5 boys to 4 girls?

6. If 62 lb. of sea-water contain 2 lb. of salt, how much salt must be added so that 42 lb. of sea-water will contain 2 lb. of salt?

Solution.—Here we see that 60 lb. of fresh water are mixed with 2 lb. of salt, and in the second condition, 40 lb. of fresh water are mixed with 2 lb. of salt. If 40 lb. are mixed with 2 lb., we find that 60 lb. will be mixed with 3 lb.; hence there were added 3 lb., minus 2 lb., or 1 lb. of salt.

7. If 50 lb. of sea-water contain 2 lb. of salt, how much salt must I add to these 50 lb. that 40 lb. of the new mixture may contain 4 lb. of salt?

8. In a mixture of gold and silver, consisting of 50 oz., there are 3 oz. of silver; how much gold must be added that there may be $\frac{1}{4}$ oz. of silver to 5 oz. of gold?

9. In a school of 60 pupils there are 20 girls; how many boys must leave the school so that there may be 10 girls to every 25 pupils?

10. A has \$20 in gold and silver, and for every \$6 of gold he has \$4 of silver; how much gold must be added that there may be \$9 of gold for \$3 of silver?

REMARK—Since the gold is to the silver as 6 to 4, and there are \$20 in all, we find there are \$12 of gold and \$8 of silver. After the addition, since 3 times the silver equals the gold, 3 times \$8, or \$24, is the gold, and \$24 — \$12, or \$12, equals the amount added.

11. A man has 40 pieces of money, consisting of copper and silver, and for every 7 of copper there are 3 of silver; how many pieces of silver must be added that for every 4 of copper there may be 2 of silver?

12. A drover has 100 animals, consisting of sheep and cows, and he has 2 sheep for every 3 cows; how many sheep must he sell that he may have 2 sheep to 6 cows?

13. There are 50 pupils in a certain school, consisting of girls and boys, and there are 8 boys to 2 girls; how many boys must leave the school that there may be 6 boys to 2 girls? *

14. A man expends 60 cents for an equal number of apples and pears, giving 3 cents each for his apples and 2 cents each for his pears; how many pears must he sell that the remainder may be to his apples as 2 to 3?

15. A tree 90 feet in length, by falling, was broken into two parts such that $\frac{1}{3}$ of the shorter equaled $\frac{1}{5}$ of the longer; how much must be cut from the longer so that $\frac{1}{4}$ of it may equal $\frac{1}{5}$ of the other part?

16. A boy bought 24 oranges and lemons, and $\frac{2}{3}$ of the number of oranges equals $\frac{2}{5}$ of the number of lemons; how many more oranges must be purchased, that $\frac{2}{5}$ of the number of oranges may equal $\frac{2}{3}$ of the number of lemons?

LESSON VI.

Coach and Cow Problems.

HOW far may a person ride in a coach, which goes at the rate of 10 miles an hour, so that he may be gone but 6 hours, provided he walks home at the rate of 5 miles an hour?

Solution.—If he goes 10 miles an hour, he will go 1 mile in $\frac{1}{10}$ of an hour; and if he returns 5 miles an hour, he will return 1 mile in $\frac{1}{5}$ of an hour; hence to go and return a mile, it takes $\frac{1}{5} + \frac{1}{10}$, or $\frac{3}{10}$ of an hour. Therefore, in 6 hours he can go and return as many miles as $\frac{3}{10}$ is contained times in 6, or 20 miles.

2. How far may a person ride in a coach, going at the rate of 9 miles an hour, provided he is gone only 10 hours, and walks back at the rate of 6 miles an hour?

3. How many miles may I sail in a steamboat, going at the rate of 15 miles an hour, provided I am gone only 9 hours, and return at the rate of 12 miles an hour?

4. A steamboat, whose rate of sailing in still water is 12 miles an hour, descends a river whose current is 4 miles an hour, and is gone 6 hours; how far did it go?

5. A boat, whose rate of sailing is 10 miles an hour, moves down a river whose current is 2 miles an hour; how far may it go that it may be gone but 5 hours?

6. Eight men hire a coach to ride to Lancaster, but by taking in 4 more persons the expense of each is diminished by $\$\frac{3}{4}$; what do they pay for the coach?

Solution.—If the expense of one is diminished $\$\frac{3}{4}$, the expense of 8 is diminished 8 times $\$\frac{3}{4}$, or \$6, which the 4 men pay. If 4 men pay \$6, 8 + 4, or 12 men, which are 3 times 4 men, will pay 3 times \$6, or \$18.

7. Ten men hire a coach for a certain sum of money, but taking in 5 more persons, the expense of each is diminished $\$\frac{1}{5}$; what did the coach cost them?

8. Twenty persons engage a pleasure-boat for sailing, but before they start 12 of the company decline going,

by which the expense of each is increased $3; what did they pay for the boat?

9. Suppose that for every 4 cows a farmer has he should plow an acre of land, and allow one acre of pasture for every 2 cows; how many cows could he keep on 15 acres?

Solution.—If for every 4 cows he plows 1 acre, for 1 cow he plows $\frac{1}{4}$ of an acre; and if for every 2 cows he pastures 1 acre, for one cow he pastures $\frac{1}{2}$ an acre; hence 1 cow requires $\frac{1}{2}+\frac{1}{4}$, or $\frac{3}{4}$ of an acre, and on 15 acres he could keep as many cows as $\frac{3}{4}$ is contained times in 15, or 20.

10. Suppose that for every 3 cows a farmer has he plows an acre of land, and allows 1 acre of pasture for every 2 cows; how many cows can he keep on 20 acres?

11. Suppose that for every 4 cows a farmer has he should plow 1 acre of land, and allow 1 acre of pasture for every 3 cows; how many cows could he keep on 140 acres?*

12. If a farmer plows 1 acre of land for every 4 cows, and allows 1 acre of pasture for every 5 cows, how many acres will it require to keep 60 cows?

13. A man keeps 72 cows on his farm, and for every 4 cows plows 1 acre, and keeps 1 acre of pasture for every 6 cows; how many acres in his farm?

14. If a farmer keeps 40 cows on 18 acres of plowed and pastured land, and plows 1 acre for every 4 cows, how many acres of pasture must he allow for 4 cows?

15. A farmer keeps 48 cows on 14 acres of plowed and pastured land; he reserves 1 acre of pasture for every 6 cows; how many acres will he plow for every 16 cows?

16. A company of 15 persons engage a dinner at a hotel, but before paying the bill 5 of the company withdraw, by which each person's bill was augmented $\$\frac{1}{2}$; what was the bill?

LESSON VII.

Miscellaneous Problems.

MARY gave Lilly 24 pins, which is $\frac{3}{5}$ of what Lilly already had, and $\frac{3}{5}$ of what Mary had remaining; how many had each at first?

2. Henry gave his sister 20 cents, which is $\frac{4}{5}$ of what he had at first more than his sister, and $\frac{1}{2}$ of what she now has; how much had each at first?

3. What is the height of a pole the shadow of which is 10 feet, at the same time that a pole 10 feet high casts a shadow $2\frac{1}{2}$ feet in length?

4. A man lost $\frac{4}{5}$ of all his money, and then won $\frac{3}{4}$ as much as he lost, and then had \$20; how much money had he at first?

5. A merchant exchanged 7 grammars for 6 arithmetics; how much were the grammars worth each, if 5 arithmetics cost 35 dimes?

6. A farmer, having sold $\frac{1}{2}$ of his sheep and 10 cows, found that 15 sheep and $\frac{1}{3}$ of his cows remained; how many of each did he own?

7. Stephen lost 12 cents, then found $\frac{1}{2}$ as much as he lost, and then had $\frac{3}{4}$ as much as he had at first; how much had he at first?

8. Maria gave away some money, and then found 10 cents, which was $\frac{1}{2}$ of what she then had, and $\frac{1}{5}$ of what she at first had; how much did she give away?

9. If to $\frac{1}{2}$ of the cost of B's horse you add \$80, the sum will equal $\frac{2}{3}$ of the cost; required the cost of his horse.

10. A paid \$80 for flour, and $\frac{3}{5}$ of the number of dollars is 3 times the number of barrels purchased; what was the price of the flour a barrel?

11. Philo found 10 cents, and then lost $\frac{2}{5}$ of what he found, and then had $\frac{3}{5}$ as much as he had at first; how much had he at first?

12. A has 8 marbles and B has 7, and 6 times what they both have is equal to the number that C has, increased by 10; how many has C?

13. A can do as much work in 2 days as B can in 4 days or C in 6 days; in how many days can B do as much as C can in 18 days?

14. A can do as much in 6 days as B can in 2 days, and B can do as much in 5 days as C can in 15 days; in how many days can A do as much as C can in 4 days?

15. A can do 3 times as much in a day as B, and B can do twice as much as C; in how many days can A do as much as C can in 4 days?

16. A farmer has 102 animals, consisting of hogs, sheep, and cows; there are $\frac{3}{4}$ as many sheep as hogs, and $\frac{4}{5}$ as many cows as hogs; required the number of each.

17. One-half of Mary's oranges equals Annie's, and $\frac{1}{2}$ of Annie's equals Emma's, and they together have 28; how many has each?

18. A earned $\frac{2}{3}$ as much as B, and B earned $\frac{3}{4}$ as much as C, and they together earned $108; required the amount earned by each.

19. Reuben had 9 oranges worth 4 cents apiece, and Jackson had 8 lemons worth 3 cents apiece, which he gave to Reuben for a part of his oranges; how many oranges had Reuben remaining?

20. A farmer had 40 sheep in one field, which was $\frac{4}{5}$ of the number in another field; then $\frac{1}{2}$ of the sheep in each field jumped into the other; how many then in each field?

21. A man sold his horse and sleigh for $200, and $\frac{4}{5}$ of this is 8 times what his sleigh cost, and the horse cost 10 times as much as the sleigh; required the cost of each.

22. Frescoln had 30 apples, and Lucy gave him 10 more; he then gave his father 7, and his mother a certain number, and had 13 remaining; how many did he give his mother?

23. A and B had each 30 apples; A gave B 10 of his, and B gave A 6 of his, and then lost a certain number, so that A had 12 more than B; how many did B lose?

24. A and B had each 40 cents; A gave B 10 of his, and B gave A twice as many of his, and then, losing a certain number, had twice as many as A gave him; how many did he lose?

25. Two times a certain number, + 10, equals 3 times the sum obtained by increasing the number by 2; what is the number?

26. A horse and cow eat a quantity of hay in 3 weeks; how long will it last each, provided the horse eats only $\frac{2}{3}$ as much as the cow?

27. Two pipes fill a cistern in 15 hours, and $\frac{2}{3}$ of what one pours in equals $\frac{2}{5}$ of what the other pours in; how long will it take each to fill it?

28. Three men, A, B, and C, can build a boat in 12 days, and their rates of working are as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; how long would it take each alone to build it?

29. The distance from Mount Joy to Harrisburg is 25 miles, and $\frac{4}{5}$ of this distance is $\frac{5}{8}$ of $\frac{4}{9}$ of the distance from Harrisburg to McVeytown; what is the distance to McVeytown?

30. John and Henry can mow 60 acres of grass in 6 weeks, and $\frac{1}{2}$ of what John can mow in a day equals what Henry can mow in a day; how long will it take each to mow it?

31. From Philadelphia to Lancaster the distance is 68 miles, and $\frac{1}{4}$ of this, increased by 2 miles, equals $\frac{4}{7}$ of the distance from Lancaster to Harrisburg, minus 1 mile; required the distance to Harrisburg.

32. B can drink a keg of mead in 4 days; $\frac{2}{3}$ of what B drinks equals $\frac{1}{2}$ of what A drinks, and also $\frac{1}{3}$ of what C drinks. After the three had been drinking $\frac{6}{13}$ of a day, A and C drink the remainder; how long did it take them?

SECTION VIII.

ADVANCED COURSE.

This Section is designed to afford a complete and thorough course in Mental Arithmetic for normal and high schools, and for advanced classes in public schools.

Teachers who wish can take a part of this course by having their classes solve the problems in each lesson as far as the one marked with a star.

LESSON I.

Cup and Cover and Borrowing Problems.

A PERSON has 2 silver cups, and only one cover for both. The first cup weighs 12 ounces. If the first cup be covered, it will weigh twice as much as the second, but if the second cup be covered, it will weigh 3 times as much as the first; required the weight of the second cup and cover.

Solution.—By the last condition of the problem, 3 times 12 ounces, or 36 ounces, equals the weight of the second cup and cover, which, added to 12 ounces, the weight of the first cup, equals 48 ounces, the weight of the two cups and the cover. By the second condition of the problem, twice the weight of the second cup equals the weight of the first cup and cover, which, added to the weight of the second cup, equals 3 times the weight of the second cup, which equals the weight of all, or 48 ounces. If 3 times the weight of the second cup equals 48 ounces, etc.

2. A gentleman has two silver cups, and only one cover for both. The first cup weighs 10 ounces; and if the first cup be covered, it will weigh 3 times as much as the second, but if the second cup be covered, it will weigh 5 times as much as the first; required the weight of the second cup and cover.

3. A lady bought two watches and a chain. The chain

and gold watch cost 4 times as much as the silver watch, and the chain and silver watch cost twice as much as the gold watch; what is the value of each, if the silver watch is worth $30?

4. A farmer bought a horse, colt, and saddle. If the horse be saddled, it will be worth 5 times as much as the colt, but if the colt be saddled, it will be worth $\frac{1}{2}$ as much as the horse; what is the value of the horse and the saddle, supposing the colt to be worth $50?

5. The head of a fish is 10 inches long; 7 times the length of the head equals the length of the body and tail, and 3 times the length of the tail equals the length of the head and body; required the length of the tail and body respectively.

6. A went to a store and borrowed as much money as he had, and spent 4 cents; he then went to another store and did the same, and then had 4 cents remaining; how much money had he at first?

7. A boy went to a store, borrowed as much money as he had, and spent 8 cents; he then went to a second store, borrowed as much as he had, and spent 12 cents, and had no money remaining; how much money had he at first?

8. Roland went to a store, borrowed as much money as he had, and spent 8 cents; he then went to a second and third store and did the same, and had no money remaining; how much had he at first?*

9. A farmer bought a cow for $30, which was $\frac{1}{3}$ of what he paid for a horse and sheep, and $\frac{1}{5}$ of what he paid for the horse and cow equals what the sheep cost; required the cost of the horse and sheep respectively.

10. Reynolds went to a hotel, borrowed as much money as he had, and spent two cents; he then went to a second and third hotel, did the same, and had 6 cents remaining; how much money had he at first?

11. William went to a store, borrowed 10 cents, and

then spent 8 cents; doing the same at a second and third store, he found he had doubled his money; how much money had he?

12. James went to a store, borrowed 10 cents, and then spent 12 cents; he did this at a second and third store, and than had no money left; how much money had he at first?

13. A man expended $5 for ducks, which was $\frac{1}{3}$ of what he paid for geese and turkeys, and twice what he paid for geese equals what he paid for ducks and turkeys; how many of each kind did he buy, provided the ducks cost $\$\frac{1}{2}$, the geese $1, and the turkeys $3 each?

LESSON II.

Chess and Dining Problems.

A AT a game of chess lost $15, and then won $\frac{1}{3}$ as much as he had remaining, and then had $\frac{1}{2}$ as much as he had at first; how much had he at first?

Solution.—After winning $\frac{1}{3}$ as much as he had remaining, he had $\frac{2}{3} + \frac{1}{3}$, or $\frac{4}{3}$, of what remained after losing $15, which equals $\frac{1}{2}$ of what he had at first; hence $\frac{3}{3}$ of what remained equals $\frac{3}{8}$ of what he had at first. Then $\frac{8}{8}$ of what he had at first, minus $\frac{3}{8}$ of what he had at first, which is $\frac{5}{8}$ of what he had at first, equals $15, etc.

2. B at a game of chess lost $22, and then won $\frac{1}{4}$ as much as he had remaining, and then had $\frac{1}{3}$ as much as he had at first; how much had he at first?

3. A, having a certain sum of money, found $20, and then lost $\frac{1}{3}$ of what he then had, and then had twice as much as he had at first; how much had he at first?

4. A man, having some money, borrowed 30 cents, and then, losing $\frac{1}{4}$ of what he then had, found there remained 3 times as much as at first; how much had he at first?

5. A boy lost 44 cents, and then, earning $\frac{2}{3}$ as much as

remained, found he had $\frac{3}{4}$ as much as at first; how much had he at first?

6. A man went to a store and spent 21 cents, and then, borrowing $\frac{1}{5}$ of what he had remaining, had $\frac{1}{2}$ as much as he had at first; how much money had he at first?

7. A at a game of chess won \$18, and then lost $\frac{3}{4}$ of what he then had, when, counting his money, he found he had $2\frac{1}{2}$ times as much as at first; how much did he make by playing?

8. A, B, and C dine together, A furnishing 2 loaves, B 3 loaves, and C contributing 25 cents to be divided between A and B; required the share of each.

9. A boy, being asked his age, replied that if my age in 3 years be diminished by its $\frac{2}{5}$, the remainder will be $\frac{3}{4}$ of my age now; what was his age?

10. A furnished 2 loaves for supper, and B 4 loaves, while C contributed 20 cents to be divided between A and B; how much of it should each receive?*

11. A and B lost 32 peaches, and then bought $\frac{2}{3}$ as many as remained, and then had $\frac{3}{5}$ as many as at first; how many had each at first, supposing their shares to be as 2 to 3?

12. A boy, being asked his age, replied that if 11 years ago his age had been increased by its $\frac{1}{4}$, it would then have been $\frac{1}{3}$ of what it now is; required his age.

13. A lady, being asked her age, said that if her age were increased by its $\frac{1}{5}$, the sum would equal 3 times her age 12 years ago; what was her age?

14. A person, being asked his age, said that if my age in 4 years be diminished by its $\frac{2}{3}$, the remainder will equal $\frac{1}{2}$ of my age 4 years ago; what was his age?

15. A furnished 6 eggs for a repast, and B 10 eggs, while C contributed 16 cents to be divided between A and B; how much shall each receive, provided A and B eat the same number, and C eats 4 more than each?

16. Two partners, A and B, lost \$210, and the next

year gained $\frac{1}{3}$ of what remained, which was $\frac{1}{6}$ of the original stock; what was the stock of each, if $\frac{2}{3}$ of A's equals $\frac{2}{5}$ of B's stock?

17. A, B, and C eat 14 peaches, of which A owned 5 and B 9, and C contributed 24 cents; how much of the money ought A and B each to receive, if B eats twice as many as A, and C eats twice as many as B?

18. Two merchants, having a certain number of yards of cloth, bought 30 yards more, then sold $\frac{1}{4}$ as many as they had, and then had 3 times as many as at first; how many yards had each, if $\frac{1}{2}$ of A's number equals $\frac{1}{3}$ of B's?

LESSON III.

Time Problems.

WHAT is the time of day, provided $\frac{1}{2}$ of the time past midnight equals the time to noon?

Solution.—By the condition of the problem, $\frac{1}{2}$ of the time past midnight equals the time to noon, which, added to $\frac{2}{3}$ of the time past midnight, equals $\frac{2}{3}$ of the time past midnight, which equals the time from midnight to noon, or 12 hours. If $\frac{2}{3}$ of the time past midnight equals 12 hours, $\frac{1}{2}$ equals $\frac{1}{3}$ of 12 hours, which is 4 hours, and $\frac{2}{3}$ equals 2 times 4, or 8 hours, the time past midnight. Hence, it is 8 o'clock in the morning.

2. What is the time of day, supposing $\frac{2}{3}$ of the time past midnight equals the time to noon?

3. What is the time of day, if $\frac{1}{3}$ of the time past midnight equals the time past noon?

4. What is the time of day, if $\frac{1}{5}$ of the time past midnight equals the time past noon?

5. What is the hour of day, supposing $\frac{1}{7}$ of the time past midnight equals the time to midnight again?

6. What is the hour of day, provided $\frac{3}{5}$ of the time past midnight equals the time to midnight again?

7. What is the hour of day, if $\frac{1}{3}$ of the time to noon equals the time past midnight?

8. A person, being asked the time of day, said, $\frac{1}{5}$ of the time past noon equals the time to midnight; what was the hour?

9. William recited his lesson when $\frac{3}{7}$ of the time past noon equaled the time past midnight; at what hour did he recite?

10. What time after 12 o'clock are the hour and minute-hands of a watch exactly together?

REMARK.—It will be seen that the minute-hand gains on the hour-hand 11 spaces in going 12; hence, to gain 1 space, the distance they are apart at 1 o'clock, it must go $\frac{1}{11}$ of 12 spaces, which is $1\frac{1}{11}$ spaces, or $5\frac{5}{11}$ minutes.

11. What time after 2 o'clock are the hour and minute-hands of a clock together?

12. A man, being asked the hour of the day, replied that it was between 4 and 5 o'clock, and that the hour and minute-hands were together; what was the time?

13. A person, being asked the time of day, said, $\frac{3}{5}$ of the time to midnight equals the time past midnight; what was the time?*

14. A lady, being asked the time of day, replied that $\frac{1}{6}$ of the time to midnight equaled $\frac{1}{2}$ of the time past noon; what was the time?

15. Required the hour of day, provided $\frac{2}{7}$ of the time to midnight equals $\frac{2}{3}$ of the time to noon.

16. One-half of the time past 9 o'clock A. M. equals $\frac{1}{3}$ of the time to midnight; what is the time?

17. A man, being asked the hour of day, replied that $\frac{1}{4}$ of the time past 3 o'clock equaled $\frac{1}{2}$ of the time to midnight; what was the hour?

18. A man, being asked the hour of day, replied that $\frac{1}{8}$ of the time past 2 o'clock equaled $\frac{1}{2}$ of the time to midnight; what was the hour?

19. What time between 7 and 8 o'clock are the hour and minute-hands of a watch exactly together?

20. In how many minutes after 4 o'clock will the hour- and minute-hands be 5 minute spaces apart?

21. In how many minutes after 4 o'clock will the hour- and minute-hands be 5 minutes of time apart?

22. A lady, being asked the hour of day, replied that $\frac{2}{3}$ of the time past noon equaled $\frac{4}{5}$ of the time to midnight, minus $\frac{4}{5}$ of an hour; what was the time?

LESSON IV.

Age Problems.

HENRY is 35 years old, and Mary is 5; in how many years will Henry be 6 times as old as Mary?

Solution.—At the required time, 6 times Mary's age will equal Henry's age; then 6 times Mary's age, which is Henry's age minus Mary's age, equals 5 times Mary's age, which equals the difference between their ages, which is $35 - 5$, or 30 years. If 5 times Mary's age equals 30 years, once her age equals $\frac{1}{5}$ of 30 years, which is 6 years. Hence, when Mary is 6 years old Henry will be 6 times as old as she, but Mary is now 5; therefore, in $6 - 5$, or 1 year, Henry will be 6 times as old as Mary.

2. James is 28 years old, and Ellen is 8; in how many years will James be 3 times as old as Ellen?

3. A is 30 years and B is 6 years old; in how many years will A be only 4 times as old as B?

4. Eva is 6 years old, and her mother 7 times as old; in how many years will the mother be 5 times as old?

5. Morton is 10 years old, and Moses 30; how long since Moses was 5 times as old as Morton?

6. Jacob is twice as old as his son, who is 20 years of age; how long since Jacob was 5 times as old as his son?

7. Mary is $\frac{1}{4}$ as old as her aunt, who is 40 years of age; how long since Mary was only $\frac{1}{7}$ as old as her aunt?

8. Jason is 5 times as old as John, and the difference

of their ages is 20 years; in how many years will Jason be 3 times as old as John?

9. Henry is 4 times as old as William, and the sum of their ages is 25 years; in how many years will Henry be but 3 times as old as William?

10. Two-thirds of A's age equals $\frac{4}{5}$ of B's age, and the difference between their ages is 10 years; how long since A was 3 times as old as B?

11. A lady bought some ribbon at 5 cents a yard, and as much more at the rate of 7 cents a yard, and sold it all at 8 cents a yard; how much did she buy if she gained 20 cents?

REMARK.—She gained 2 cents on 1 yard; hence, to gain 20 cents, there were $20 \div 2$, or 10 yards.

12. Albert bought some oranges at 2 cents each, and as many more at 4 cents each, and sold them at the rate of 2 for 8 cents, and gained 12 cents; required the number bought.

13. Witmer bought some tea at 4 francs a pound, and as much more at 8 francs, and sold it all for 7 francs a pound, and gained 40 francs; required the number of pounds of each kind.

14. Bowman mixed some sugar worth 5 cents a pound with an equal quantity worth 9 cents a pound, and sold the mixture for 10 cents a pound, and gained \$6; required the quantity of each kind.*

15. One-half of M's age equals $\frac{1}{3}$ of N's age, and the difference of their ages is 10 years; in how many years will $\frac{1}{3}$ of M's age equal $\frac{1}{4}$ of N's age?

16. Two-tenths of B's age equals $\frac{4}{5}$ of C's age, and the sum of their ages is 30 years; in how many years will B be 3 times as old as C?

17. Two-third's of Daniel's age equals $\frac{3}{4}$ of David's age, and the sum of their ages is 68 years; how long since $\frac{2}{3}$ of Daniel's age equaled $\frac{4}{5}$ of David's age?

18. Sophia bought a number of yards of silk at the

rate of 3 yards for $1, and as much more at the rate of 4 yards for $1, and sold it all at the rate of 8 yards for $3, and thereby gained $5; how many yards did she buy?

19. A boy bought some apples at the rate of 4 for 1 cent, and as many more at the rate of 5 for 1 cent, and sold them all at the rate of 10 for 2 cents, and thereby lost 5 cents; how many of each kind did he buy?

20. A boy bought some apples at 2 cents apiece, and twice as many oranges at 4 cents apiece, and sold them all at 3 cents each, and thereby lost $1; how many of each kind did he buy?

LESSON V.

Age and Step Problems.

A'S age equals 4 times B's, but in 5 years A's age will be only 3 times B's; how old is each?

Solution.—By the first condition, 4 times B's age equals A's age, hence, the difference of their ages is 3 times B's age, and once B's age equals $\frac{1}{3}$ of the *difference* of their ages: in 5 years, 3 times B's age equals A's age, hence, twice B's age equals the *difference* of their ages, and once B's age then equals $\frac{1}{2}$ of the *difference*. Therefore, 5 years is the difference between $\frac{1}{2}$ of *the difference* and $\frac{1}{3}$ of *the difference* of their ages, or $\frac{1}{6}$ of *the difference* of their ages; and $\frac{6}{6}$, or *the difference* of their ages, is 6 times 5, or 30 years. If 3 times B's age equals 30 years, B's age is $\frac{1}{3}$ of 30 years, or 10 years, and A's age is 4 times 10, or 40 years.

2. John is 5 times as old as Oliver, but in 8 years he will be only 3 times as old; what is the age of each?

3. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be $\frac{1}{2}$ as old; what is the age of each?

4. Henry is $\frac{1}{5}$ as old as his father, but in 25 years he will be $\frac{3}{5}$ as old; required the age of each.

5. Ten years ago, when I first met Mr. Morgan, I was $\frac{1}{4}$ as old as he, but now I am $\frac{1}{2}$ as old as he is; required each of our ages?

6. Sixteen years ago, when Agnew married, he was 3

times as old as his wife, but now he is only twice as old; what is the age of each?

7. A hare is 30 rods before a hound. and runs 3 rods while the hound runs 6; how many rods must the hound run to catch the hare?

8. Stephen is 40 steps before James, and takes 5 steps to James's 7; how many steps must James take to catch Stephen, supposing their steps to be equal?

9. A hare takes 2 leaps while a hound takes 1, but 1 of the hound's leaps equals 4 of the hare's; how much does the hound gain on the hare in taking one leap?

10. A hare is 30 leaps before a hound, and takes 4 leaps while the hound takes 2, but 2 of the hounds's leaps equal 8 of the hare's; how many leaps must the hound take to catch the hare?

11. A fox is 40 leaps before a hound, and takes 3 leaps while the hound takes 2, but 2 of the hound's leaps equal 4 of the fox's; in how many leaps will the hound catch the fox?

12. A thief is 20 steps before an officer, and takes 6 steps while the officer takes 5, but 5 of the officer's steps equal 8 of the thief's; how far will the thief run before he is overtaken?*

13. A rabbit is 60 leaps before a hound, and takes 9 leaps while the hound takes 3, but 2 of the hound's leaps equal 7 of the rabbit's; how many leaps will the rabbit take before being caught?

14. Twenty-five years ago Willard was $\frac{1}{7}$ as old as his uncle, but 5 years ago he was $\frac{1}{3}$ as old; how old is each at present?

15. Four years ago B's house was four times as old as his barn, but 2 years hence it will be only twice as old; how long has each been built?

16. Three years ago Emma's doll was only $\frac{1}{5}$ of the age of herself, but 7 years hence it will be $\frac{3}{5}$ of her age; required the age of each?

17. B takes 30 steps to overtake C; how far was C ahead of B when they started, provided B takes 2 steps while C takes 3, and 2 of B's equal 5 of C's steps?

18. E takes 60 steps before he is overtaken by D; how many steps does D take to catch E, provided E takes 4 steps while D takes 3, and 5 of D's equal 8 of E's, and how far ahead was E when they started?

19. M and N are 60 rods apart, and approach each other; how far will each travel before they meet, provided M takes 3 steps while N takes 6, and 2 of M's equal 6 of N's steps?

20. A and B are 150 of B's steps apart, and approach each other; how many steps will each take before they are together, if 4 of A's steps equal 8 of B's, and B takes 9 steps while A takes 3?

LESSON VI.

Partnership Problems.

TWO men, A and B, in partnership gain \$300. A owns $\frac{2}{3}$ of the stock, lacking \$40, and gains \$180; required the whole stock and share of each.

Solution.—If A had owned $\frac{2}{3}$ of the stock, his gain would have been $\frac{2}{3}$ of \$300, or \$200; but he gained only \$180, therefore the \$40 must gain the difference between \$200 and \$180, which is \$20. If \$40 gain \$20, to gain \$1 it will require $\frac{1}{20}$ of \$40, which is \$2, and to gain \$300 it will require 300 times \$2, which are \$600.

2. A and B enter into partnership and gain \$240. A owns $\frac{3}{4}$ of the stock, lacking \$10, and gains \$175; required the whole stock and share of each.

3. Two men in partnership gain \$200. The first owns $\frac{1}{4}$ of the stock + \$40, and his gain is \$60; what is the entire stock and share of each?

4. A and B bought a lottery ticket with which they drew a prize of \$600. A paid $\frac{2}{3}$ of the price of the ticket,

lacking $12, and his share of the prize was $340; what did each pay for the ticket?

5. A man and his son agree to mow a certain field for $72. The son mowed $\frac{1}{4}$ of the whole, + 10 acres, and received $38; how many acres did each mow?

6. Two men pay $120 for the pasturage of some cattle. The first turns in $\frac{1}{4}$ of the whole number, + 20, and pays $40; how many cattle does each turn in?

7. A farmer bought a certain number of sheep for $60; had he bought 5 more at $1 less each, they would have cost him $75; how many sheep did he buy?

Solution.—The 5 more, at $1 less each, cost $75 — $60, or $15, and one, at this rate, cost $\frac{1}{5}$ of $15, or $3, which, increased by $1, equals $4, the price of those purchased; hence, there were as many purchased as $4 is contained times in $60, which are 15.

8. A farmer bought a certain number of cows for $200; had he bought 2 more at $2 less each, they would have cost $216; how many did he buy?

9. Mr. A bought a number of turkeys for $5; had he bought 3 times as many, + 4, for the same price, they would have cost him $12 more; how many did he buy?

10. A teacher bought a number of books for $8; had he bought 4 times this number, lacking 5, they would have cost $4 more; how many did he buy?*

11. A man bought a certain number of sheep for $80; if he then buys twice as many more, at $2 less each, they all will cost $180; how many did he buy?

12. A and B agree to mow a field of grass for $60; A mows twice as much as B, lacking 8 acres, and receives $24; how many acres does each mow?

13. Two men engage to build a boat for $84; the first labors as many days as the second, + 6 days, and receives $48; how many days does each labor?

14. Two men receive the same sum for labor; but had one received $10 more and the other $6 less, one would

have received 5 times as much as the other; how many does each receive?

15. Said James to Isaac, Our purses contain the same sum of money; but if you give me \$20, and I give you \$10, I shall have 3 times as much as you; how much money had each?

16. A man, having an equal number of cows in two fields, sold $\frac{1}{3}$ of the number from each, then, 7 having jumped from the first into the second, there were 3 times as many in the second as in the first; required the number in each field.

17. Two men, A and B, agree to dig a ditch for \$50; and $\frac{4}{5}$ of what A digs, increased by 4 rods, equals $\frac{3}{4}$ of what B digs, and B receives \$30; how many rods did each dig?

LESSON VII.

Animal Problems.

A MAN bought a number of sheep for \$100, when, a dog having killed 8 of them, he sold $\frac{1}{3}$ of the remainder for cost, and received \$20; how many did he buy?

Solution.—If $\frac{1}{3}$ of the remainder cost \$20, $\frac{3}{3}$ of the remainder cost 3 times \$20, or \$60; then, since they all cost \$100, the 8 must have cost \$100 — \$60, or \$40, etc.

2. A farmer bought a number of pigs for \$80, when, 5 of them having died, he sold $\frac{2}{3}$ of the remainder for cost, and received \$40; how many did he buy?

3. A lady purchased a number of yards of muslin for \$1.50, and after using 6 yards she sold $\frac{2}{3}$ of the remainder for cost, and received 90 cents less than it all cost; how many yards did she purchase?

4. A bought a number of turkeys for \$10, when, having killed 10, he sold $\frac{2}{5}$ of the remainder for cost, re-

ceiving $8 less than the cost of all; required the number purchased.

5. A bought a number of sheep for a certain sum; and having lost 6, he sold $\frac{3}{4}$ of the remainder for cost, and received $15, which was $35 less than they all cost; how many did he buy?

6. A farmer bought a number of hens for a certain sum; and having killed 10, he sold $\frac{4}{5}$ of the remainder for cost, and received 48 dimes, which was 72 dimes less than they all cost; how many did he retain?

7. A bought a number of ducks for $16; and having killed 12, he sold $\frac{4}{5}$ of the remainder, lacking 8, for cost, and received $4; how many did he buy?

8. B bought a number of sheep for $30; and losing 2, he sold $\frac{3}{4}$ of the remainder, lacking 3, for cost, and received $21 less than all cost; required the number bought.

9. Henry bought a number of pigs for $48; and losing 3 of them, he sold $\frac{2}{3}$ of the remainder, minus 2, for cost, receiving $32 less than all cost; required the number purchased.

10. A bought some calves for $80; and having lost 10, he sold 4 more than $\frac{2}{3}$ of the remainder for cost, and received $32 less than all cost; required the number purchased.*

11. A dog killed $\frac{1}{4}$ of A's sheep; now, if he sells the remainder for cost, he will receive $60; but reserving 8 and selling $\frac{1}{2}$ of the remainder for cost, he will receive $22; how many had he at first?

12. A lost $\frac{2}{5}$ of his hens, and found if he sold $\frac{2}{3}$ of the remainder for cost, he would receive 40 dimes, but if he kept 15 and sold $\frac{2}{3}$ of the remainder, he would receive 20 dimes; how many did he have?

13. B lost $\frac{2}{3}$ of his turkeys, and then finds that by selling $\frac{2}{3}$ of the remainder for cost, he would receive

$20; but finding 6, and selling $\frac{2}{3}$ of the number he then had, he received $24; how many did he retain?

14. A lost $\frac{3}{4}$ of his hens; now, if he finds 10 and sells $\frac{3}{4}$ of his number then for cost, he will receive 60 dimes; but if he loses 10 and sells $\frac{3}{4}$ of the remainder for cost he will receive 30 dimes; how many had he at first?

15. B lost $\frac{2}{3}$ of his sheep; now, if he finds 5, and sells $\frac{3}{5}$ of what he then has for cost price, he will receive $18; but if he loses 5, and sells $\frac{3}{5}$ of the remainder for cost price, he will receive $6; how many had he at first?

LESSON VIII.

Involution and Evolution Problems.

THE square of a certain number is 64; what is the number?

Solution.—If the square of a number equals 64, the number equals the square root of 64, which is 8.

2. A boy being asked his age, replied, 3 times the square of my age equals 75 years; how old was he?

3. Albert, being asked how many marbles he had, answered, $\frac{1}{2}$ of the square of the number equals 18; how many marbles had he?

4. Three-fourths of the square of the number of letters in a sentence equals 27; how many letters are there in the sentence?

5. The square of twice a number equals 256; what is that number, and what is the square of $\frac{1}{2}$ of the number?

Remark.—The square root of 256, which is 16, equals twice the number, etc. Or, the square of *twice* a number equals *four* times the square of the number, etc.

6. If $\frac{2}{5}$ of the number of trees in an orchard be squared, the result will be 100; how many trees are there in the orchard?

7. The square of twice a number is 18 more than twice the square of the number; what is the number?

8. Twice the square of a number is 8 more than 6 times the square of half the number; what is the number?

9. Three-fourths of the square of a number is 36 more than $\frac{3}{4}$ of the square of half the number; required the number?

10. Fifteen is 3 more than $\frac{3}{2}$ of the cube of a number; what is that number?

11. Two-thirds of the cube of a number is 10 more than the cube of $\frac{2}{3}$ of the number; what is the number?

12. Two-thirds of the square of twice a number is equal to $\frac{4}{3}$ of the square of $\frac{3}{2}$ of the number, diminished by 3; what is the number?*

13. A boy spends $\frac{1}{2}$ of his money, + $\$\frac{1}{2}$, then $\frac{1}{2}$ of the remainder, + $\$\frac{1}{2}$, and then had \$3; how much money had he at first?

14. C and D together have 20 sheep, and $\frac{1}{3}$ of C's number, + $\frac{1}{4}$ of D's, equals $\frac{1}{2}$ of C's; how many sheep does each own?

15. A man spent $\frac{1}{2}$ of his money and \$2 more, and then spent \$2 more than $\frac{1}{2}$ of the remainder, and then had \$2 remaining; required his money at first?

16. Sarah gave away $\frac{1}{3}$ of her peaches, lacking $\frac{1}{3}$ of a peach, and then gave away $\frac{1}{3}$ of the remainder, lacking $\frac{1}{3}$ of a peach, and then had $5\frac{1}{3}$ peaches remaining; how many peaches had she at first?

17. A fish caught in the Conowingo weighs 8 pounds, and $\frac{2}{5}$ of the body, + $\frac{2}{3}$ of the head and tail, weigh as much as $\frac{4}{5}$ of the body; required the weight of each part if the tail is $\frac{1}{3}$ as heavy as the head.

18. A lady, being asked how many music-pupils she had, replied, $\frac{2}{3}$ of the number multiplied by $\frac{3}{4}$ of the number is 9 more than the square of $\frac{1}{2}$ the number; how many had she?

LESSON IX.

Will Problems.

A FATHER willed $4400 to his two children, A and B, whose ages were 11 and 16 years respectively, in such a manner that the parts, at 5 per cent., simple interest, would amount to equal sums when they became of age; what were the parts?

Solution.—A's money was on interest 10 years and B's 5 years. For 10 years at 5 per cent., $\frac{1}{2}$ of the principal equals the amount; hence, $\frac{1}{2}$ of A's *share* equals A's *amount;* and in the same way we see that $\frac{1}{4}$ of B's *share* equal's B's *amount.* Now, the amounts are to be equal; hence, $\frac{1}{2}$ of A's share $= \frac{1}{4}$ of B's share, from which we find A's share $= \frac{1}{2}$ of B's share; then $\frac{6}{6}$ of B's $+ \frac{2}{6}$ of B's, or $\frac{8}{6}$ of B's share $= \$4400$, $\frac{1}{6}$ of B's $= \$400$, $\frac{6}{6}$ of B's $= \$2400$, and $\frac{6}{6}$ of B's, or A's $= \$2000$.

2. A man left $5600 to his two sons, whose ages were 11 and 15 years, in such a manner that the two parts on interest, at 5 per cent., would amount to equal sums when they became 21 years of age; required the parts.

3. A gentleman, dying, divided $5100 among his three sons, whose ages were 9, 11, and 17 respectively, so that the different shares, being on interest at 5 per cent., would amount to equal sums when they became of age; what were the shares?

4. A widow divided $3700 among her sons, whose ages were respectively 14, 16, and 18 years, in such a manner that the shares, being put on interest at 20 per cent., would amount to equal sums when they became of age; required the share of each.

5. A man left $26,000 to his wife, son, and daughter, on condition that if the daughter died before becoming of age the widow should have $\frac{1}{4}$ of the fortune, but if the son died the widow should have $\frac{3}{4}$ of it; required the share of each if they all live.

REMARK.—It will be readily seen that the widow has three times as much as the daughter, and the son three times as much as the widow, which is 9 times as much as the daughter.

6. A man, wishing to erect some buildings, concluded that if he built a store and a barn the store should cost $\frac{1}{3}$ of his money, but if he built a house and a barn, the barn should cost $\frac{1}{3}$ of his money; what was the cost of each, supposing he built all three, and their cost was $77,000?

7. A father left $5500 to his son and daughter, whose ages are respectively 19 and 15 years, so that, being on interest at 10 per cent., the son should receive twice as much as the daughter when they were 21 years of age; what was the share of each?*

8. Divide $290 between A and B, whose ages are respectively 15 and 19 years, in such a manner that the parts, being placed on interest at 10 per cent., shall amount to such sums, at the time they are 21, that $\frac{3}{4}$ of A's shall be equal to $\frac{2}{5}$ of B's money.

9. A man, having a daughter in France and a son in Spain, willed, if the daughter returned, and not the son, the wife should have $\frac{2}{3}$ of the fortune, but if the son returned, and not the daughter, he should have $\frac{2}{3}$ of the fortune; they both returned, and it was found that the son received $3000 more than the daughter; required the fortune and share of each.

10. A, B, and C, contemplating the purchase of a farm, agreed if A and B bought it, A should pay $\frac{2}{5}$ of the price, but if B and C bought it, B should pay $\frac{2}{5}$ of the price; at length the three agreed to buy it together, when it was found that C paid $500 more than A; what did the farm cost, and what did each pay?

LESSON X.

Miscellaneous Problems.

BARTON pays $\frac{1}{4}$ of his salary for board; what per cent. does he have left for other purposes?

2. If I sell $\frac{1}{5}$ of a quantity of grain, and $\frac{1}{2}$ of the remainder is spoiled, what per cent. remains?

3. Bought apples at 6 cents for 4, and sold them at the rate of 6 for 4 cents; what was the loss per cent.?

4. If William's money is 25 per cent. less than Henry's, how many per cent. is Henry's money greater than William's?

5. I gave 20 per cent. of my money to A, 25 per cent. of the remainder to B, and had $30 remaining; how much money had I at first?

6. A sold B a gun and gained 25 per cent., and B sold it to C for $24 and gained 20 per cent.; what did A pay for it?

7. B bought melons for 20 per cent. more than 10 cents each, and sold them for 20 per cent. less than 10 cents each; required the loss per cent.

8. If a book is bought for $\frac{3}{4}$ of its value, and sold for 20 per cent. more than its value, what is the gain per cent.?

9. A lost $60 by selling a horse for 30 per cent. less than its cost; required the cost and the amount received.

10. Henry received $224 to invest in property, after retaining 12 per cent. on the amount invested; how much did he invest?

11. B bought goods 20 per cent. below par, and sold them 20 per cent. above par; supposing he gained $90, what amount of goods did he buy?

12. A man bought goods 25 per cent. below par, and sold them 20 per cent. above par; how much did he invest if he gained $270?

13. A merchant asked for cloth 20 per cent. more than cost, but sold it for $\frac{1}{2}$ of his asking price; what was the loss per cent.?

14. B asked for flour 25 per cent. more than cost, but sold it for 80 per cent. of the price asked; what did he lose per cent?

15. Janson sold 20 per cent. of his apples to A, 25 per cent. of the remainder to B, and $33\frac{1}{3}$ per cent. of this last remainder to C, and had 20 barrels remaining; how many had he at first?

16. A grocer asked for sugar 20 per cent. more than cost, and sold it for $33\frac{1}{3}$ per cent. less than he asked for it; what was the loss per cent?

17. A man asked 10 per cent. less for an article than cost, but sold it for $33\frac{1}{3}$ per cent. more than he asked for it; required the gain per cent.

18. What must I ask for hay, worth \$10 a ton, that, after falling 20 per cent., I may gain 20 per cent. on the value?

19. What must I charge for flour, worth \$5 a barrel, that, after falling 25 per cent. on the price, I may gain 20 per cent. on the cost?

20. What must I ask for cloth, worth \$40, that, after falling 20 per cent. on the price, I may gain 30 per cent. on the cost?

21. If my retail gain is 25 per cent., and my wholesale gain is 20 per cent. of my retail less, what per cent. do I gain at wholesale?

22. If I retail at a gain of 50 per cent., and sell at wholesale for 25 per cent. less than at retail, what do I gain per cent. at wholesale?

23. If my gain at retail is 60 per cent., and my gain at wholesale is 25 per cent. of my retail gain less, what is my gain per cent. at wholesale?

24. If A's gain at wholesale is 20 per cent., and his

gain at retail is 25 per cent. of his wholesale more, what does he gain per cent. at retail?

25. If B's loss at wholesale was 10 per cent., and his retail price was $33\frac{1}{3}$ per cent. more, what was the gain per cent. by retail?

26. A lost 40 per cent. of his flour, and sold the remainder at a gain of 50 per cent.; did he gain or lose, and how much per cent.?

27. A barrel of molasses lost 20 per cent. by leakage, and the remainder was sold at a gain of 40 per cent.; required the gain per cent.

28. An article lost 25 per cent. by wastage, and the remainder was sold for 20 per cent. above cost; what per cent. was gained or lost?

29. A drover lost $33\frac{1}{3}$ per cent. of his cattle, and sold the remainder at a gain of 50 per cent.; required the gain or loss per cent.

30. The amount of B's fortune for 4 years, at 10 per cent., is \$200 more than its amount for 6 years at 5 per cent.; required the fortune.

31. The amount of C's fortune for 2 years, at 10 per cent., is \$400 less than its amount for 6 years at 5 per cent.; what is the fortune?

32. The amount of M's money for 5 years, at 8 per cent., is \$40 more than its amount for 4 years at 6 per cent.; required his money.

33. The amount of D's money for 2 years, at 5 per cent., is \$60 more than its interest for 9 years at 10 per cent.; what is his money?

34. The amount of B's fortune for 5 years, at 10 per cent., is \$330 more than the amount of C's for the same time and rate per cent.; what is the fortune of each, provided that B's is twice C's?

LESSON XI.

Miscellaneous Problems.

A HAS 3 times as many plums as B, and B has twice as many as C; how many has each, if A has 12 more than B and C together?

2. A, having his fortune on interest at 5 per cent., in one year spends $\frac{1}{2}$ of his income in traveling, $\frac{1}{3}$ for educational purposes, and saves $100; what is his fortune?

3. A can do a piece of work in 20 days, B and C in 12 days, and if all work 6 days, C can complete it in 3 days; in what time could B and C each have done it?

4. B gained 50 per cent. in each of 3 years on what he had at the beginning of the year, and then had gained $190; what was his first capital?

5. Edward and Ella have $900, and 20 per cent. of Edward's money equals 25 per cent. of Ella's money; how much money has each?

6. Three-fifths of A's age was his wife's age when married, but in 40 years $\frac{4}{5}$ of his age equals hers; what was the age of each when married?

7. A boat whose rate of sailing is 5 miles an hour moves down a river whose current is 3 miles an hour; how far may it go that it may be back in 10 hours?

8. A and B, in partnership, gain $40; A owned $\frac{4}{5}$ of the stock, lacking $12, and B's share of the gain was $10; required the whole stock and share of each.

9. A digs $\frac{2}{3}$ of a ditch in 8 days, and then, calling in B, they together finish it in 9 days; in what time could B have done it alone?

10. A man bought hay for $8 a ton, but in getting it he lost 20 per cent. of it; what did it really cost him a ton?

11. I bought 8 apples for 16 cents, and lost $\frac{3}{4}$ of them; what per cent. must I gain on the remainder that I may neither gain nor lose by the transaction?

12. A bought 9 melons for 36 cents; but losing $33\frac{1}{3}$ per cent. of them, how must the remainder be sold to gain $33\frac{1}{3}$ per cent. by the transaction?

13. B lost 20 per cent. of his marbles; what must he gain per cent. on the remainder, that he may gain 20 per cent. on the whole?

14. A barrel of molasses leaked away 20 per cent.; what per cent. must I gain on the remainder that I may gain 40 per cent. by the transaction?

15. A sum of money at interest amounts, in 2 years, to \$240, and in 6 years to \$320; required the sum and rate per cent.

16. The amount of a sum of money for 3 years is \$230, and the amount for 4 times as long, at $\frac{1}{2}$ the same rate, is \$260; what are the sum and rates per cent.?

17. The amount of A's fortune for 3 years, at 10 per cent., is \$520 more than the amount of B's for 5 years at 6 per cent.; required the fortune of each, supposing A's to equal 3 times B's.

18. A man sold 2 horses for \$250; on one he gained 25 per cent.; on the other he lost 20 per cent.; did he gain or lose, and how much, if he received for the second $\frac{3}{5}$ as much as for the first?

19. A man sold 2 horses for \$210; on one he gained 25 per cent., and on the other he lost 25 per cent.; how much did he gain, supposing the second horse cost him $\frac{3}{5}$ as much as the first horse?

20. A man sold a horse and carriage for \$230; on the horse he lost 20 per cent., and on the carriage he gained 25 per cent.; did he gain or lose, and how much, if $\frac{4}{5}$ of what he paid for the horse equaled $\frac{3}{5}$ of the cost of the carriage?

LESSON XII.

Miscellaneous Problems.

A PERSON, being asked the hour of the day, replied that 2 hours ago the time past noon was $\frac{1}{3}$ of the time to midnight 2 hours hence; required the time.

2. A man went to a store and spent 20 cents, and then, losing $\frac{3}{4}$ as much as remained, had $\frac{1}{3}$ as much as he had at first, minus \$1; how much had he at first?

3. A is 10 steps before B, and takes 2 steps while B takes 4, and 4 of A's steps equal 6 of B's; how many steps will each take before they are together?

4. Said E to F, my age is 5 years more than yours, but 4 years ago my age was $\frac{1}{2}$ of what yours will be 4 years hence; what was the age of each?

5. A lady bought 10 yards of silk at the rate of \$4 a yard, but finding some of it damaged, for it she only paid \$1 a yard, and thus paid \$28; how many yards were damaged?

6. C and D ran from the same point in the same direction, and when D had run 40 rods, $\frac{1}{3}$ of the distance C had run equaled the distance he was ahead of D; how much did C, in running 40 rods, gain on D?

7. A boy bought some peaches at 4 cents each, and 3 times as many pears at 2 cents each, and sold them all at 6 cents each, and thus gained 28 cents; how many of each did he buy?

8. My friend's watch loses 2 minutes in 3 hours, and mine gains 20 minutes a day; they were set by correct time yesterday noon, and are now half an hour apart; what time is it?

9. A, B, and C can mow a field in 20 days, A and B in 30 days, and B and C in 40 days; after the three had worked 5 days, A and C finished it; in what time was it completed?

10. A person, being asked the time of day, replied

that $\frac{2}{3}$ of the time past midnight, 2 hours ago, equaled $\frac{2}{5}$ of the time to midnight, 3 hours and 20 minutes hence; required the time.

11. A man left $5000 to his wife, son, and daughter, so that if the daughter died before becoming of age the widow should have $\frac{1}{3}$ of the fortune, but if the son died she should have $\frac{2}{3}$ of it; required the shares of the son and daughter if the widow dies.

12. A pole whose length was 44 feet was broken into two unequal parts; if the shorter be increased by 3 feet, and the longer be diminished by 5 feet, the first will be $\frac{1}{2}$ as long as the second; required the length of each part.

13. A staff whose length is 33 feet is in the air and water; and the length in the air, -2 feet, equals 4 times the length in the water, $+6$ feet; required the length in the air.

14. Two years ago Mr. Smith was 5 times as old as his son John will be 2 years hence, and 3 years hence his age will equal 15 times John's age 3 years ago; required the age of each.

15. Harry gave $\frac{1}{4}$ of his money, lacking 3 cents, to James, $\frac{1}{3}$ of the remainder, lacking 2 cents, to Willie, and $\frac{1}{2}$ of the remainder, lacking 1 cent, to Charles, and then had 8 cents remaining; what was Harry's money before his gifts?

16. Jordan gave $\frac{1}{5}$ of his money, plus 4 cents, to John, $\frac{1}{4}$ of the remainder, plus 3 cents, to George, and $\frac{1}{3}$ of what now remained, plus 2 cents, to Jackson, and found he had $\frac{1}{5}$ as much as at first; how much money had he at first?

17. A lady, having two watches, bought a chain for $20. If the chain be put on the silver watch, their value will be $\frac{1}{3}$ as much as the gold watch; but if it be put on the gold watch, they will be worth 7 times as much as the silver watch; what was the value of each watch?

18. Two boats leave a wharf at the same time for the same point; it takes one boat 10 hours to reach it, and the other boat sails 5 times as fast going, and 10 times as fast returning; when do they meet?

19. Paid $34\frac{1}{2}$ for corn at $\$\frac{3}{4}$, wheat at $1, and oats at $\$\frac{1}{2}$ a bushel; sold $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat for 50 per cent. advance, gaining on the corn $\frac{2}{3}$ as much as on the wheat, and on the sale the cost of the oats; how much of each did I buy?

20. A person has two cups and a cover which weighs 30 ounces. If the first cup be covered, it will weigh twice as much as the second, but if the second cup be covered, it will weigh 3 times as much as the first; what is the weight of each cup?

21. A and B were engaged by a Pennsylvania farmer to dig 100 rods of ditch for $100; and since the part which A was to dig was more difficult of excavation than that which B dug, it was agreed that A should receive 10 shillings per rod, and B 6 shillings per rod. They each received $50 for their labor. How many rods did each dig?

REMARK.—In some States $1 equals 8 shillings, in others, 7s. 6d., etc.

NOTES.—1. For additional problems, see "KEY TO THE NEW MENTAL ARITHMETIC AND METHODS OF TEACHING MENTAL ARITHMETIC, etc.," by the author of this work, where may be found a large collection, of an amusing and interesting character, under the head of *Social Arithmetic*.

2. In this "KEY" will also be found a short treatise on *Mental Algebra*, which may be used in connection with the arithmetical solutions of the mental arithmetic by any teachers who may desire to do so.

THE END.

Date Due